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10MAT21

Second Semester B.E. Degree Examination, June 2012
Engineering Mathematics - II

Time: 3 hrs.

Max. Marks:100

- Note:** 1. Answer FIVE full questions choosing at least two from each part.
 2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
 3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1. a. Select the correct answer : (04 Marks)
 - i) We say that the given differential equation is solvable for x, if it is possible to express x in terms of
 A) x and y B) x and p C) y and p D) x, y and p
 - ii) The general solution of $P^2 - 7P + 12 = 0$ is
 A) $(y + 3x - c)(y + 4x - c) = 0$ B) $(y - 3x - c)(y - 4x - c) = 0$
 C) $(y - 4x)(y + 3x) = 0$ D) None of these
 - iii) The general solution of the equation $y = 3x + \log P$ is
 A) $y = 3x + 3 + c e^y$ B) $y = 3x + \log(3 + c e^y)$
 C) $y + 3x = 3 + c e^y$ D) None of these
 - iv) The general solution of the equation $(y - Px)^2 = 4P^2 + 9$ is
 A) $y = c x + \sqrt{4c^2 + 9}$ B) $y = c + \sqrt{4c^2 + 9}$
 C) $y = c x + \sqrt{4c^2 - 9}$ D) $y - c x = 4 c^2 + 9$
- b. Solve : $p^2 + 2py \cot x = y^2$. (05 Marks)
- c. Solve : $p^2 + 4x^5p - 12x^4y = 0$, obtain the singular solution also. (05 Marks)
- d. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$. (06 Marks)
2. a. Select the correct answer : (04 Marks)
 - i) P.I. of $y'' - 3y' + 2y = 12$ is
 A) 6 B) $y = c_1 e^x + c_2 e^{2x}$ C) $\frac{1}{12}$ D) $\frac{1}{6}$
 - ii) The complementary function of $(D^4 - a^4)y = 0$ is
 A) $y = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos x + c_4 \sin x$
 B) $y = c_1 e^{-ax} + c_2 e^{ax}$
 C) $y = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos ax + c_4 \sin ax$
 D) None of these
 - iii) If $F(D) = D^2 + 5$, $\frac{1}{f(D)} \sin 2x = \dots\dots$
 A) $\frac{-\cos 2x}{2}$ B) $\frac{\cos 2x}{2}$ C) $\sin 2x$ D) $\cos 2x$
 - iv) The solution of the differential equation $y'' - 3y' + 2y = e^{3x}$ is
 A) $y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{2} e^{3x}$ B) $y = c_1 e^x + c_2 e^{2x} + \frac{1}{2} e^{3x}$
 C) $y = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{2} e^{3x}$ D) $y = c_1 e^{-x} + c_2 e^{2x} + \frac{1}{2} e^{-3x}$

- b. Solve : $(D - 2)^2 y = 8 (e^{2x} + \sin 2x)$. (05 Marks)
 c. Solve : $y'' - 2y' + y = x \cos x$. (05 Marks)
 d. Solve $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$, given that $x = 1$, $y = 0$ at $t = 0$. (06 Marks)

3. a. Select the correct answer : (04 Marks)

- i) The Wronskian of x and e^x is
 A) $e^x (1-x)$ B) $x e^x$ C) $e^{-x} (x-1)$ D) $e^x (x-1)$
 ii) In the equation $\frac{dx}{dt} + y = \sin t + 1$, $\frac{dy}{dt} + x = \cos t$, if $y = \sin t + 1 + e^{-t}$, then $x = \dots$
 A) 0 B) e^{-t} C) $x e^{-t}$ D) e^t
 iii) In homogeneous linear differential equation whose auxiliary equation has roots 1, -1 is
 A) $y'' + y = 0$ B) $x^2 y'' - xy' - y = 0$
 C) $x^2 y'' + xy' - y = 0$ D) $y'' - y' = 0$
 iv) The solution of $x^2 y'' + xy' = 0$ is
 A) $y = c_1 + c_2 \log x$ B) $y = a \log x + 6$ C) $y = e^t$ D) $y = e^{-t}$
 b. Using the method of variation of parameters solve $y'' + 4y = \tan 2x$. (05 Marks)
 c. Solve : $(1+x)^2 y'' + (1+x) y' + y = 2 \sin [\log (1+x)]$. (05 Marks)
 d. Solve by Frobenius method, the equation

$$4x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 0. \quad (06 \text{ Marks})$$

4. a. Select the correct answer : (04 Marks)

- i) The solution of $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$ is
 A) $z = -x^2 \sin(xy) + y f(x) + \phi(x)$ B) $z = \frac{\cos(xy)}{x^2} + y f(x) + \phi(x)$
 C) $z = -\frac{\sin(xy)}{x^2} + y f(x) + \phi(x)$ D) None of these
 ii) A solution of $(y-z)p + (z-x)q = x - y$ is
 A) $x^2 + y^2 + z^2 = f(x + y + z)$ B) $x^2 - y^2 - z^2 = f(x - y + z)$
 C) $x^2 - y^2 - z^2 = f(x - y - z)$ D) None of these
 iii) The partial differential equation obtained from $z = ax + by + ab$ by eliminating a and b is
 A) $z = px + qy$ B) $z = px + qy + pq$
 C) $z = px + qy - pq$ D) $z = px - qy - pq$
 iv) The partial differential equation obtained from $z = f(x + y) + g(x - y)$ by eliminating the arbitrary functions is
 A) $r + t = 0$ B) $r - t = 0$ C) $r - a^2 t = 0$ D) $r + a^2 t = 0$
 b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. (05 Marks)
 c. Solve : $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (05 Marks)
 d. Solve by the method of variables $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0, y) = 2 e^{5y}$. (06 Marks)

PART - B

5. a. Select the correct answer :

(04 Marks)

i) The value of $\int_1^2 \int_1^3 x y^2 dx dy$ is _____

- A) 0 B) 1 C) $\frac{13}{2}$ D) 13

ii) The integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ after changing the order of integration is

- A) $\int_0^\infty \int_0^y \frac{e^{-y}}{y} dx dy$ B) $\int_0^\infty \int_y^\infty \frac{e^{-y}}{y} dx dy$
 C) $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y} dx dy$ D) $\int_0^\infty \int_0^x \frac{e^{-y}}{y} dx dy$

iii) $B\left(\frac{1}{2}, \frac{1}{2}\right) = \dots\dots$

- A) $\sqrt{\pi}$ B) $\frac{\sqrt{\pi}}{2}$ C) 3.1416 D) $-\pi$

iv) In terms of Beta function $\int_0^{\pi/2} \sin^7 \theta \sqrt{\cos \theta} d\theta = \dots\dots\dots$

- A) $\beta(4, \frac{3}{4})$ B) $\frac{1}{2} \beta(4, \frac{3}{4})$ C) $\beta(2, \frac{3}{2})$ D) $\frac{1}{2} \beta(2, \frac{3}{2})$

b. Change the order of integration in $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dx dy$ and hence evaluate the same.

(05 Marks)

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$.

(05 Marks)

d. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$.

(06 Marks)

6. a. Select the correct answer :

(04 Marks)

i) In Green's theorem in the plane $\oint_C m dx + n dy = \dots\dots\dots$

- A) $\iint_R \left(\frac{\partial m}{\partial y} + \frac{\partial n}{\partial x} \right) dx dy$ B) $\iint_R \left(\frac{\partial m}{\partial y} - \frac{\partial n}{\partial x} \right) dx dy$
 C) $\iint_R \left(\frac{\partial n}{\partial x} - \frac{\partial m}{\partial y} \right) dx dy$ D) $\iint_S \vec{F} \cdot \hat{n} ds$

ii) The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by employing Green's theorem is

- A) 0 B) 1 C) π D) $\pi a b$

iii) A necessary and sufficient condition that the line integral $\oint_L \vec{F} \cdot d\vec{R}$ for every closed curve C is

- A) $\text{curl } \vec{F} = 0$ B) $\text{div } \vec{F} = 0$ C) $\text{curl } \vec{F} \neq 0$ D) $\text{div } \vec{F} \neq 0$

- iv) If V is the volume bounded by a surface S and \vec{F} is continuously differentiable vector function then $\iiint_V \operatorname{div} \vec{F} \, dv = \dots$
- A) $\oint_e \vec{F} \cdot d\vec{r}$ B) $\iint_s \vec{F} \cdot \hat{n} \, ds$ C) $\iint_s (\nabla \times \vec{F}) \cdot \hat{n} \, ds$ D) None of these
- b. If $\vec{F} = 2x y \, \mathbf{i} + yz^2 \, \mathbf{j} + xz \, \mathbf{k}$ and s is the rectangular parallelepiped bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 1$, $z = 3$, evaluate $\iint_s \vec{F} \cdot \hat{n} \, ds$. (05 Marks)
- c. Using Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x \, dy]$, where C is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$. (05 Marks)
- d. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\mathbf{i} - 2xy \, \mathbf{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. (06 Marks)
7. a. Select the correct answer : (04 Marks)
- i) $L \{e^{2(t-1)}\} = \dots$
- A) $\frac{1}{s-2}$ B) $\frac{e^{-2}}{s-2}$ C) $\frac{1}{s+2}$ D) $\frac{e^{-2}}{s+2}$
- ii) $L \{t^{1/2}\} = \dots$
- A) $\frac{\sqrt{\pi}}{s^{3/2}}$ B) $\frac{2\sqrt{\pi}}{s^{3/2}}$ C) $\frac{\sqrt{\pi}}{2\sqrt{s}}$ D) $\frac{\sqrt{\pi}}{2s^{3/2}}$
- iii) $L \left\{ \frac{\sin t}{t} \right\} = \dots$
- A) $\frac{\pi}{2} + \tan^{-1} s$ B) $\frac{\pi}{2} - \cot^{-1} s$ C) $\cot^{-1} s$ D) $\tan^{-1} s$
- iv) $L \{ \delta(t+2) \} = \dots$
- A) e^{-as} B) e^{2s} C) e^{-2s} D) e^{as}
- b. Find the value of $\int_0^\infty t^3 e^{-t} \sin t \, dt$ using Laplace transforms. (05 Marks)
- c. Draw the graph of the periodic function $f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ and find its Laplace transform. (05 Marks)
- d. Prove that $L \{ \delta(t-a) \} = e^{-as}$. (06 Marks)
8. a. Select the correct answer : (04 Marks)
- i) $L^{-1} \left\{ \frac{1}{4s^2 - 36} \right\} = \dots$
- A) $\frac{1}{4} \cos h \, 6t$ B) $\frac{1}{12} \sin 3t$ C) $\frac{1}{6} \cos h \, 3t$ D) $\frac{1}{12} \sin h \, 3t$
- ii) $L^{-1} \left\{ \frac{1 + e^{-3s}}{s^2} \right\} = \dots$
- A) $t + (t-3)u(t-3)$ B) $(t-3)u(t-3)$
 C) $t - (t-3)u(t-3)$ D) $t + (t+3)u(t+3)$

iii) $L^{-1} \left\{ \cot^{-1} \frac{s}{a} \right\} = \dots\dots\dots$

- A) $\frac{\sin t}{t}$ B) $\frac{\sin a t}{t}$ C) $\frac{\sin h a t}{t}$ D) $\frac{\sinh t}{t}$

iv) $L \left[\int_0^t f(u) g(t-u) du \right] = \dots\dots\dots$

- A) $f(t) g(t)$ B) $f(s) g(s)$ C) $f(s) - g(s)$ D) $\frac{f(s)}{g(s)}$

b. Find $L^{-1} \left\{ \frac{5s+3}{(s-1)(s^2+2s+5)} \right\}$. (05 Marks)

c. Apply convolution theorem to evaluate

$L^{-1} \left\{ \frac{s^2}{(s^2+a^2)(s^2+b^2)} \right\}$. (05 Marks)

d. Solve $(D^3 - 3D^2 + 3D - 1) y = t^2 e^t$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = -2$ by Laplace transform method. (06 Marks)
