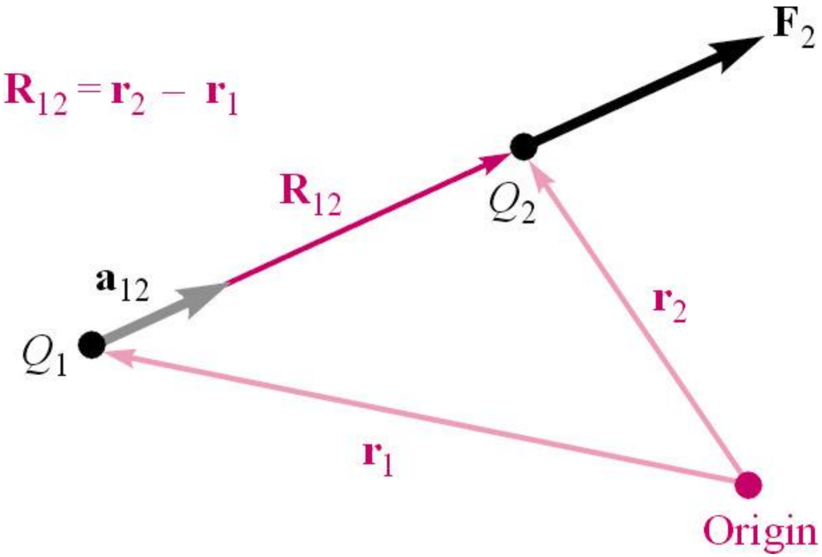
Chapter 2 - Coulomb’s Law and Electric Field Intensity

The Experimental Law of Coulomb

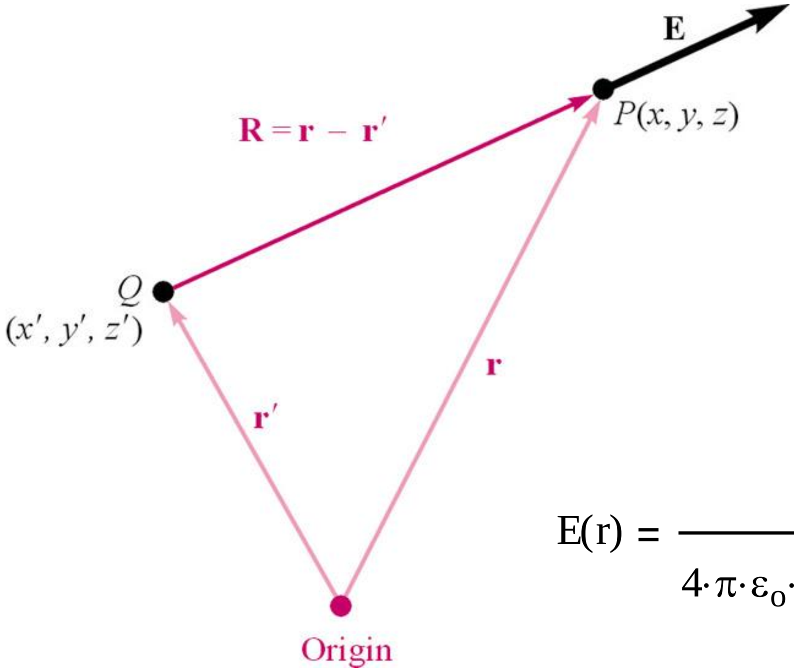
$F = k \cdot \frac{Q1 \cdot Q2}{R^2}$ $\quad k = \frac{1}{4 \cdot \pi \cdot \varepsilon_0}$ $\quad \varepsilon_0 = 8.85410^{-12} = \frac{1}{36\pi} \cdot 10^{-9} \ \frac{F}{m}$ $\longrightarrow$ $F = \frac{Q1 \cdot Q2}{4 \cdot \pi \cdot \varepsilon_0 \cdot R^2}$



$$F = \frac{Q1 \cdot Q2}{4 \cdot \pi \cdot \varepsilon_0 \cdot R^2} \cdot a_{12} \qquad a_{12} = \frac{r2 - r1}{|r2 - r1|}$$

Electric Field Intensity

$$F_t = \frac{Q1 \cdot Qt}{4 \cdot \pi \cdot \varepsilon_0 \cdot (R_{1t})^2} \cdot a_{1t}$$



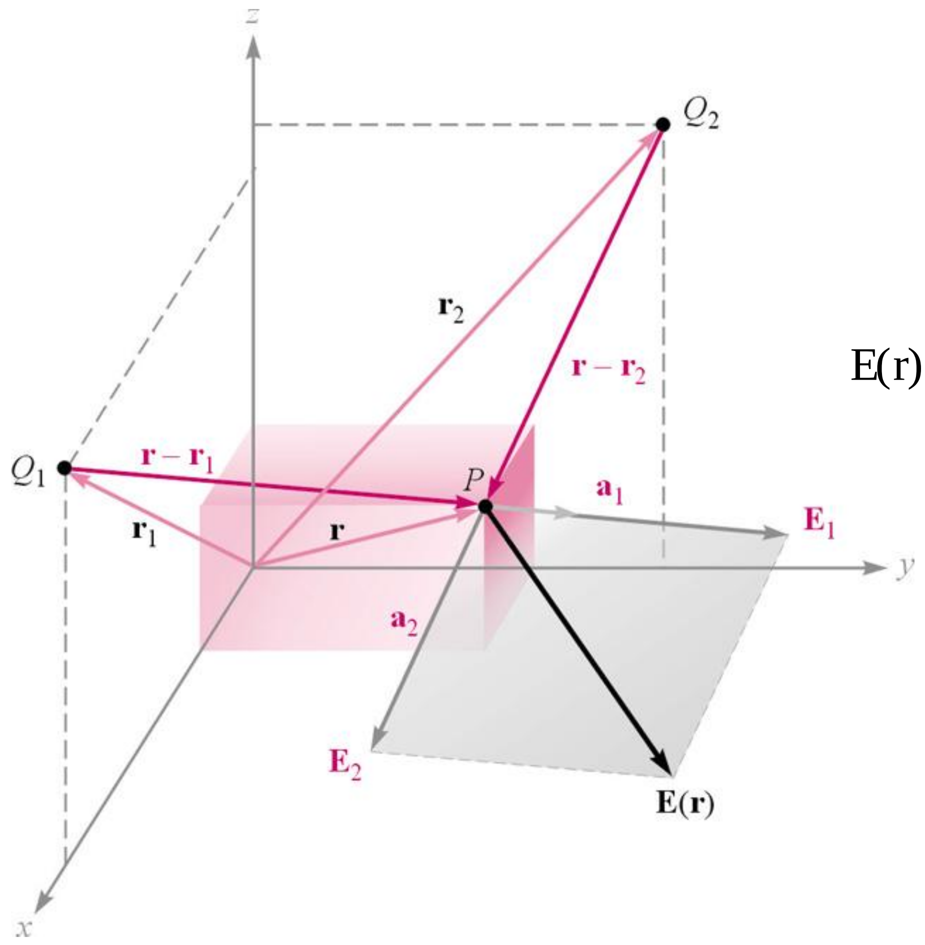
$$\frac{F_t}{Qt} = \frac{Q1}{4 \cdot \pi \cdot \varepsilon_0 \cdot (R_{1t})^2} \cdot a_{1t}$$

$$E = \frac{F_t}{Qt}$$

$$E(r) = \frac{Q}{4 \cdot \pi \cdot \varepsilon_o \cdot (|r - r1|)^2} \cdot \frac{r - r1}{|r - r1|}$$

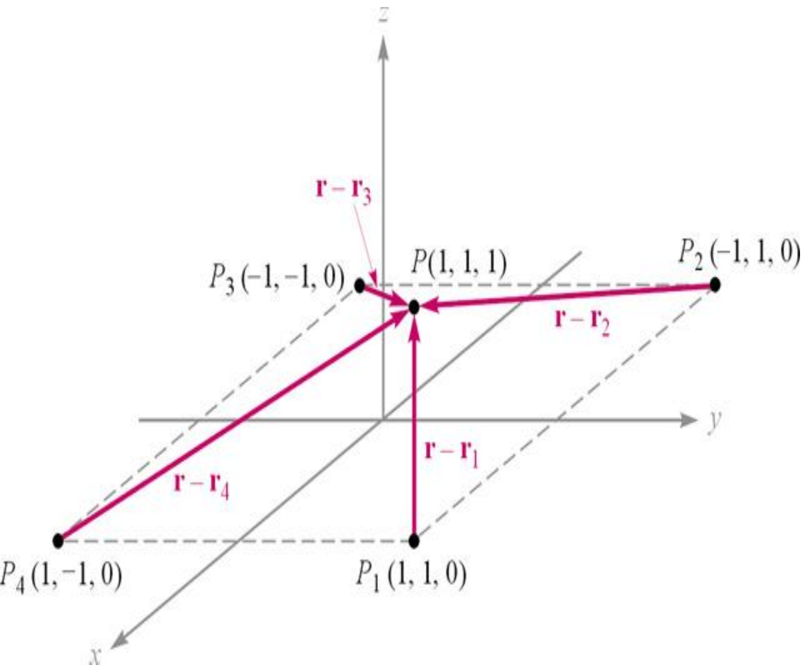
$$E(r) = \frac{Q \cdot \left[(x - x1) \cdot a_x + (y - y1) \cdot a_y + (z - z1) \cdot a_z \right]}{4 \cdot \pi \cdot \varepsilon_o \cdot \left[(x - x1)^2 + (y - y1)^2 + (z - z1)^2 \right]^{\frac{3}{2}}}$$

Electric Field Intensity



$$E(r) = \sum_{m=1}^{n} \frac{Qm}{4\cdot\pi\cdot\varepsilon_0\cdot\left(\left|r - r_m\right|\right)^2}\cdot a_m$$

Electric Field Intensity – Example 2.2



$$P := \begin{pmatrix}1\\1\\1\end{pmatrix} \quad P1 := \begin{pmatrix}1\\1\\0\end{pmatrix} \quad P2 := \begin{pmatrix}-1\\1\\0\end{pmatrix} \quad P3 := \begin{pmatrix}-1\\-1\\0\end{pmatrix} \quad P4 := \begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$Q := 3\cdot 10^{-9} \qquad \varepsilon_0 := 8.85410^{-12}$$

$$R1 := |P1 - P| \qquad R1 = 1$$

$$R2 := |P2 - P| \qquad R2 = 2.236$$

$$R3 := |P3 - P| \qquad R3 = 3$$

$$R4 := |P4 - P| \qquad R4 = 2.236$$

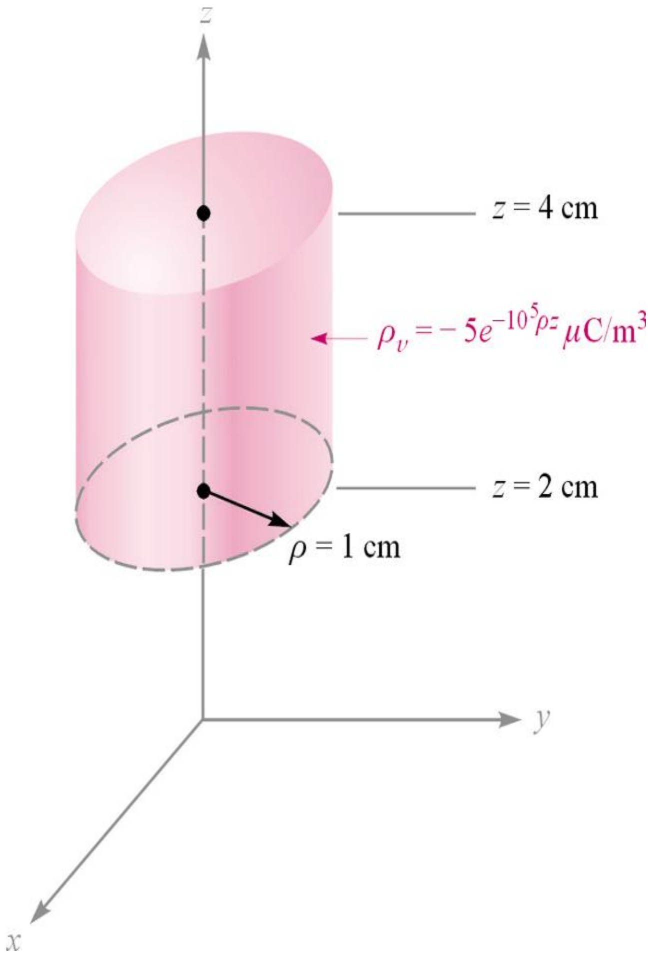
$$\frac{Q}{4\cdot\pi\cdot\varepsilon_0} = 26.963 \qquad V\cdot m$$

$$P - P1 = \begin{pmatrix}0\\0\\1\end{pmatrix} \quad P - P2 = \begin{pmatrix}2\\0\\1\end{pmatrix} \quad P - P3 = \begin{pmatrix}2\\2\\1\end{pmatrix} \quad P - P4 = \begin{pmatrix}0\\2\\1\end{pmatrix}$$

$$E := \frac{Q}{4\cdot\pi\cdot\varepsilon_0}\cdot\left(\frac{P-P1}{R1}\cdot\frac{1}{R1^2} + \frac{P-P2}{R2}\cdot\frac{1}{R2^2} + \frac{P-P3}{R3}\cdot\frac{1}{R3^2} + \frac{P-P4}{R4}\cdot\frac{1}{R4^2}\right)$$

$$E = \begin{pmatrix}6.821\\6.821\\32.785\end{pmatrix}$$

Field Due To A Continuous Volume Charge Distribution



$$Q = \int\int\int \rho v\, dx\, dy\, dz$$

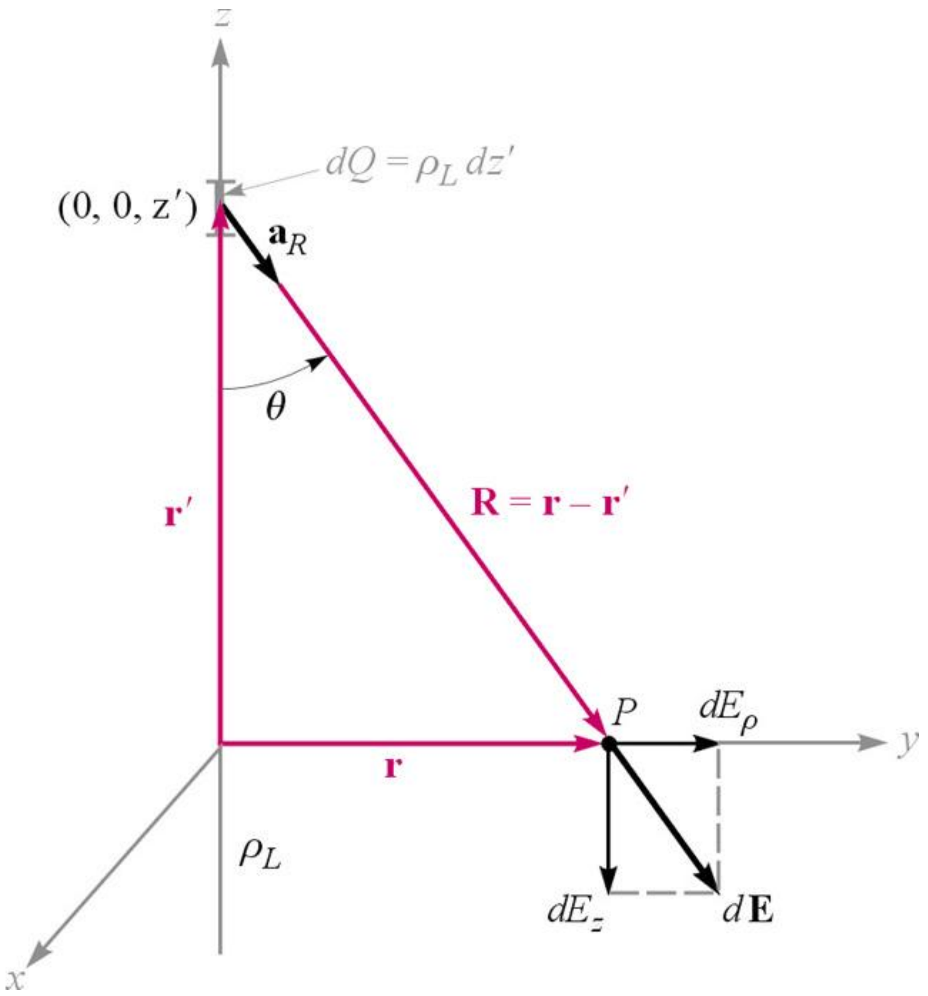
Example 2.3

$$\rho v(\rho, z) := -5 \cdot 10^{-6} \cdot e^{-10^5 \cdot \rho \cdot z}$$

$$Q := \int_{0.02}^{0.04} \int_{0}^{2\cdot\pi} \int_{0}^{0.01} \rho v(\rho, z) \cdot \rho \, d\rho \, d\phi \, dz$$

$$Q = -7.854 \times 10^{-14}$$

Field of a Line Charge

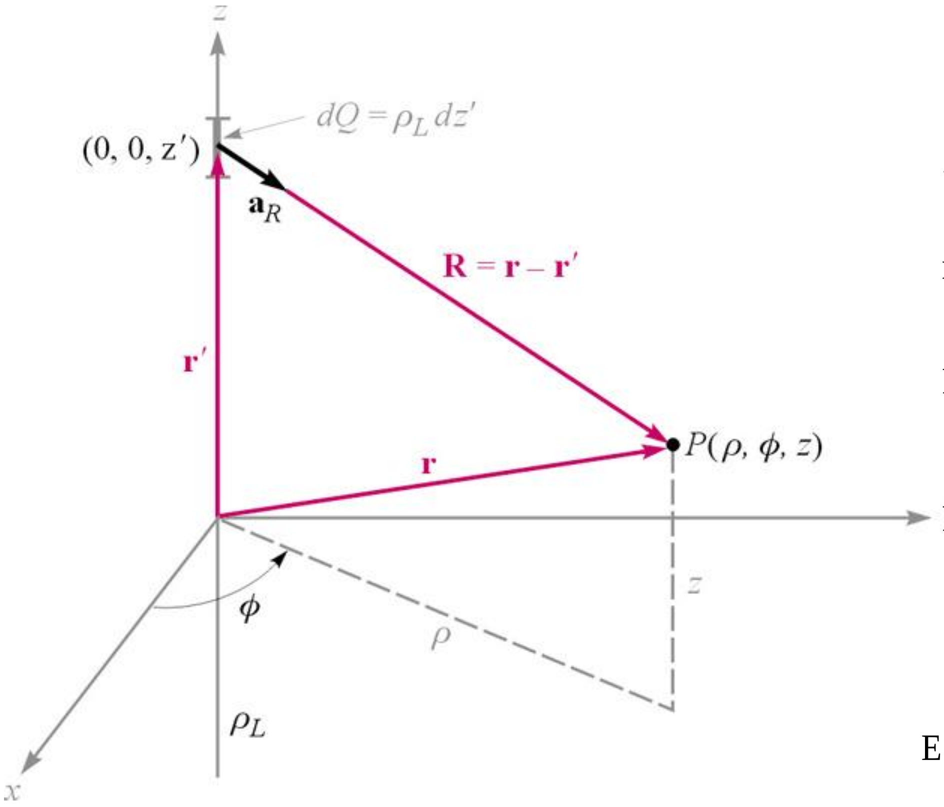


$$E\rho = \int_{-\Omega}^{\Omega} \frac{\rho L \cdot \rho}{4 \cdot \pi \cdot \varepsilon_0 \cdot \left(\rho^2 + z^2\right)^{\frac{3}{2}}} \, dz$$

$$E\rho = \frac{\rho L}{2 \cdot \pi \cdot \varepsilon_0 \cdot \rho}$$

$$E = \frac{\rho L}{2 \cdot \pi \cdot \varepsilon_0 \cdot \rho} \cdot a_\rho$$

Field of a Line Charge (neglect symmetry)



$$E = \int\int\int \frac{\rho vq}{4\cdot\pi\cdot\varepsilon 0}\cdot\frac{(r - r1)}{(|r - r1|)^3}\,dx1\,dy1\,dz1$$

$$\rho v1 = \rho L\cdot dz1$$

$$r = \rho\cdot a\rho + z\cdot az \qquad r1 = z1\cdot az$$

$$R = r - r1 = \rho\cdot a\rho + (z - z1)\cdot az$$

$$R = \sqrt{\rho^2 + (z - z1)^2} \qquad aR = \frac{\rho\cdot a\rho + (z - z1)\cdot az}{\sqrt{\rho^2 + (z - z1)^2}}$$

$$E = \int_{-\Omega}^{\Omega} \frac{(\rho L\cdot dz1)\cdot\left[\rho\cdot a\rho + (z - z1)\cdot az\right]}{\left[4\cdot\pi\cdot\varepsilon 0\cdot\left[\rho^2 + (z - z1)^2\right]\right]^{\frac{3}{2}}}\,dz1$$

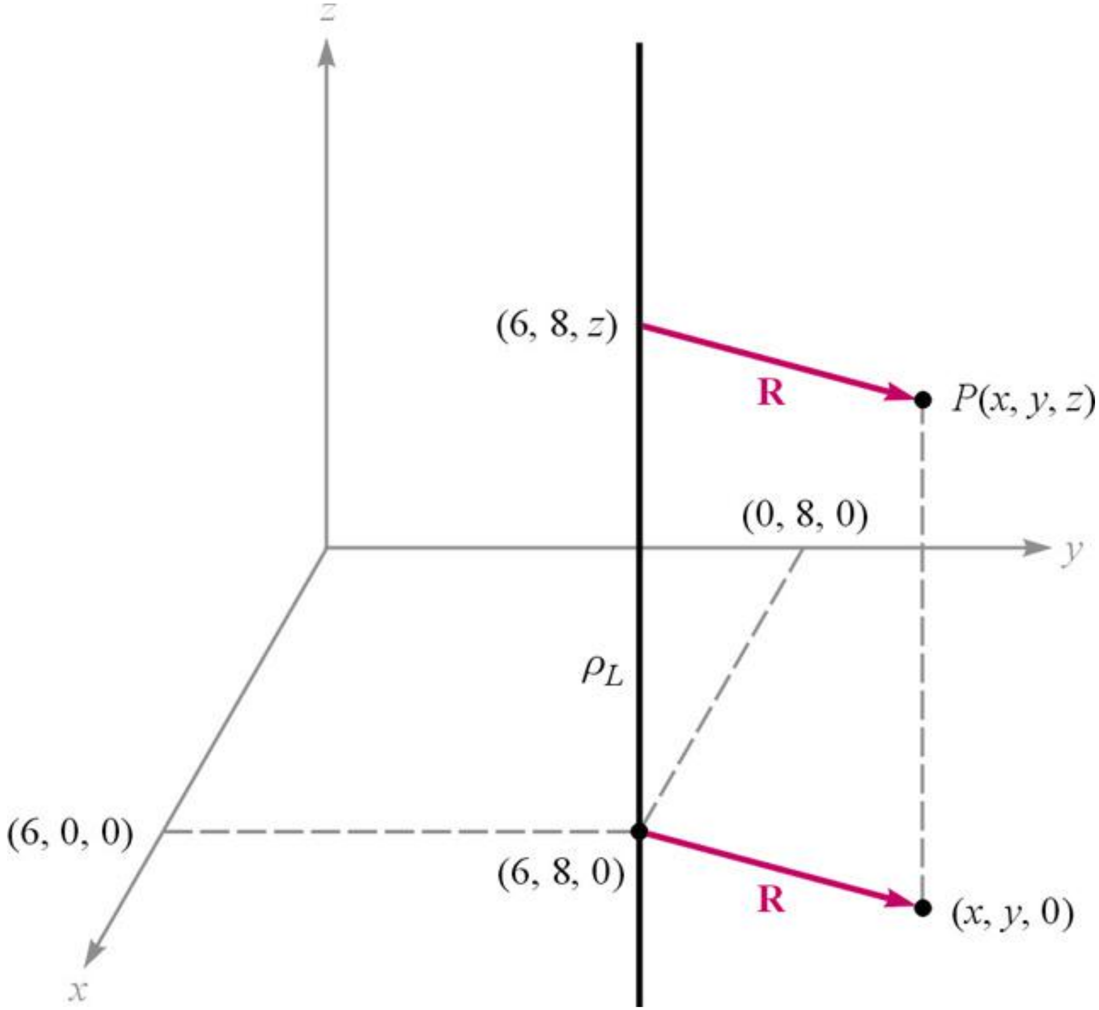
Field of a Line Charge (neglect symmetry)

$$E = \frac{\rho L}{(4\cdot\pi\cdot\varepsilon 0)}\cdot\left[a\rho\cdot\int_{-\Omega}^{\Omega}\frac{(\rho\cdot dz1)}{\left[\left[\rho^{2}+(z-z1)^{2}\right]\right]^{\frac{3}{2}}}\,dz1 + az\cdot\int_{-\Omega}^{\Omega}\frac{((z-z1))}{\left[\left[\rho^{2}+(z-z1)^{2}\right]\right]^{\frac{3}{2}}}\,dz1\right]$$

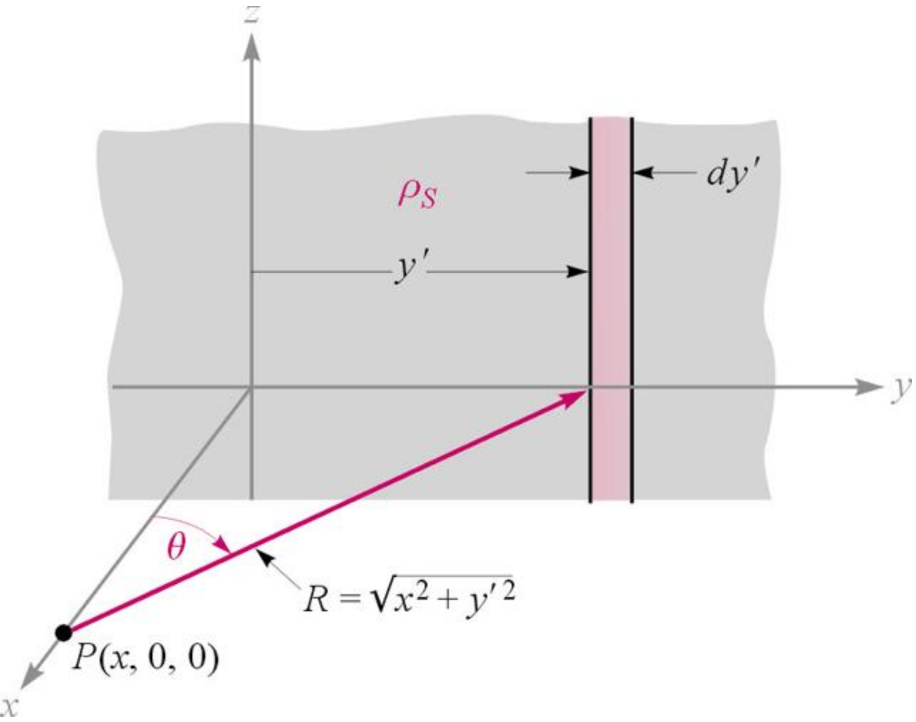
$$E = \frac{\rho L}{(4\cdot\pi\cdot\varepsilon 0)}\cdot\left[a\rho\cdot\rho\cdot\frac{1}{\rho^{2}}\cdot\overset{-\Omega\ \text{to}\,\Omega}{\frac{-(z-z1)}{\sqrt{\rho^{2}+(z-z1)^{2}}}} + az\cdot\overset{-\Omega\ \text{to}\,\Omega}{\frac{1}{\sqrt{\rho^{2}+(z-z1)^{2}}}}\right]$$

$$E = \frac{\rho L}{(4\cdot\pi\cdot\varepsilon 0)}\cdot\left(a\rho\cdot\frac{2}{\rho} + az\cdot 0\right) = \frac{\rho L}{(2\cdot\pi\cdot\varepsilon 0)\cdot\rho}\cdot a\rho$$

Field of a Line Charge



Field of a Sheet of Charge



$$R = \sqrt{x^2 + y^2}$$

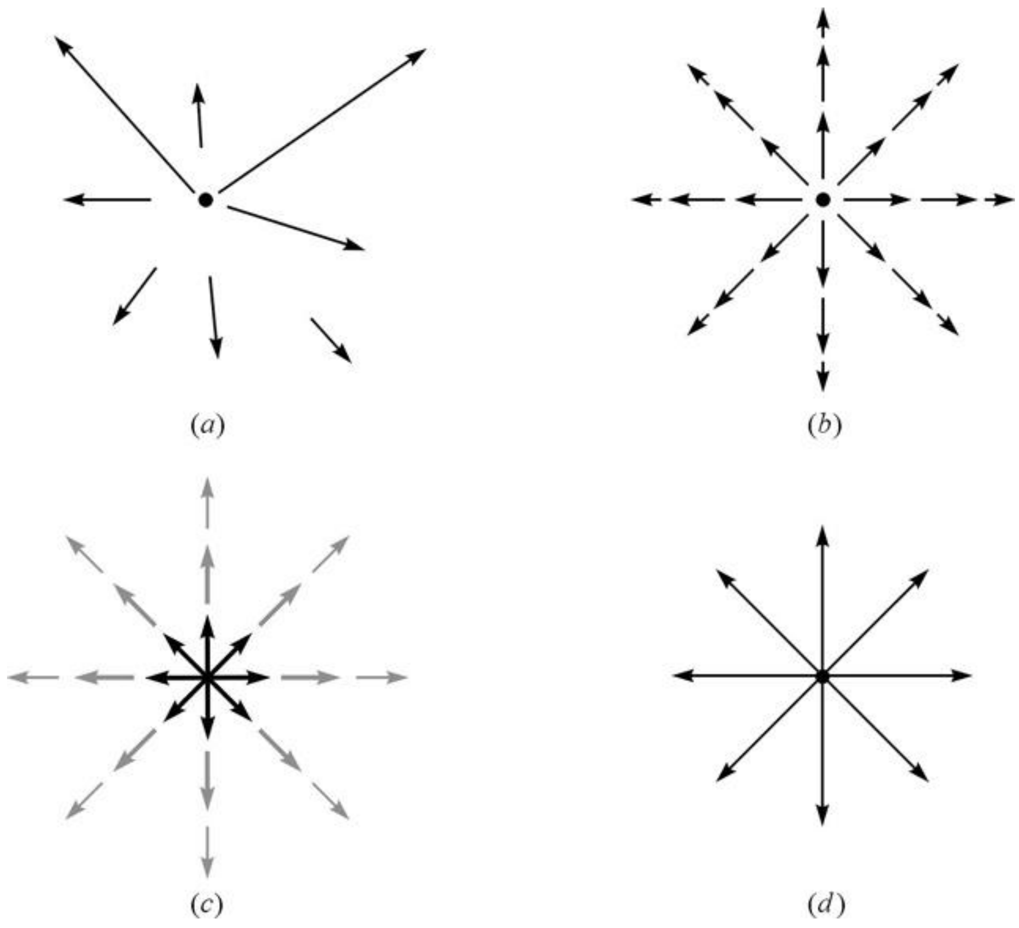
$$dEx = \rho s \cdot \frac{dy1}{2 \cdot \pi \cdot \varepsilon 0 \cdot \sqrt{x^2 + y^2}} \cdot \cos(\theta) = \frac{\rho s}{(2 \cdot \pi \cdot \varepsilon 0)} \cdot \frac{x \cdot dy1}{x^2 + y^2}$$

$$Ex = \frac{\rho s}{(2 \cdot \pi \cdot \varepsilon 0)} \cdot \int_{-\Omega}^{\Omega} \frac{x}{x^2 + y^2}\, dy1 = \frac{\rho s}{(2 \cdot \pi \cdot \varepsilon 0)} \cdot \operatorname{atan}\left(\frac{y1}{x}\right) \quad -\Omega \text{ to } \Omega$$

$$Ex = \frac{\rho s}{2 \cdot \varepsilon 0} \qquad E = \frac{\rho s}{2 \cdot \varepsilon 0} \cdot aN$$

This is a very interesting result. The field is constant in magnitude and direction. It is as strong a million miles away from the sheet as it is right of the surface.

Streamlines and Sketches of Fields

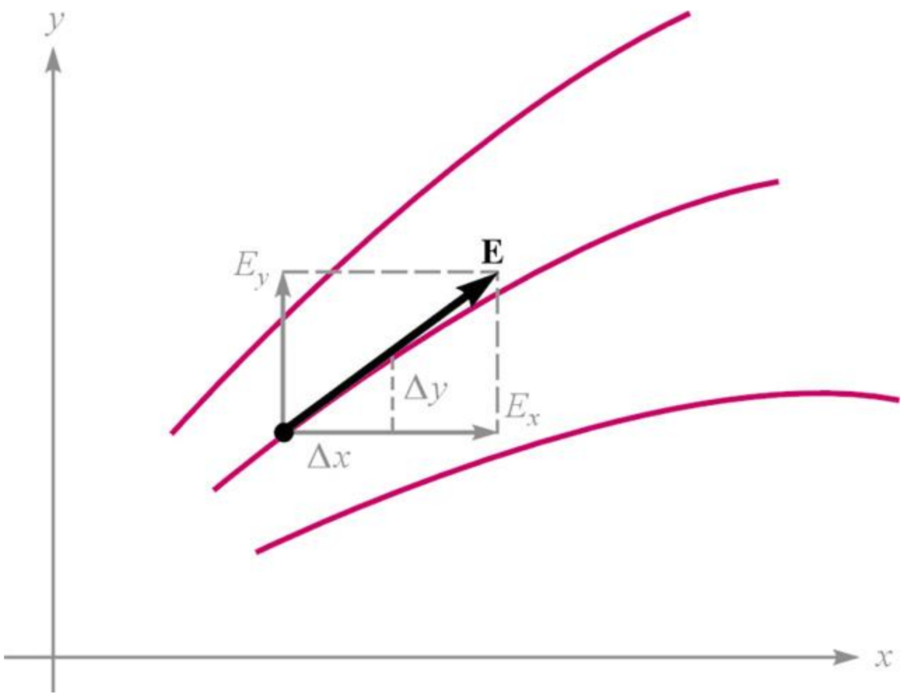


Cross-sectional view of the line charge.

Lengths proportional to the magnitudes of E and pointing in the direction of E

Streamlines and Sketches of Fields

$$\frac{Ey}{Ex} = \frac{dy}{dz}$$



$$E = \frac{1}{\rho} \cdot a\rho \qquad E = \frac{x}{x^2 + y^2} \cdot ax + \frac{y}{x^2 + y^2} \cdot ay$$

$$\frac{dy}{dx} = \frac{Ey}{Ex} = \frac{y}{x} \qquad \frac{dy}{y} = \frac{dx}{x}$$

$$\ln(y) = \ln(x) + C1 \qquad \ln(y) = \ln(x) + \ln(c)$$

$$y = C \cdot x$$

$$E(x,y) = 5 \cdot x^3 \cdot ax - 15 \cdot x^2 \cdot y \cdot ay \qquad P := \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

$$\frac{dy}{dx} = \frac{Ey}{Ex} = \frac{-15 \cdot x^2 \cdot y}{5 \cdot x^3} = -3 \cdot \frac{y}{x}$$

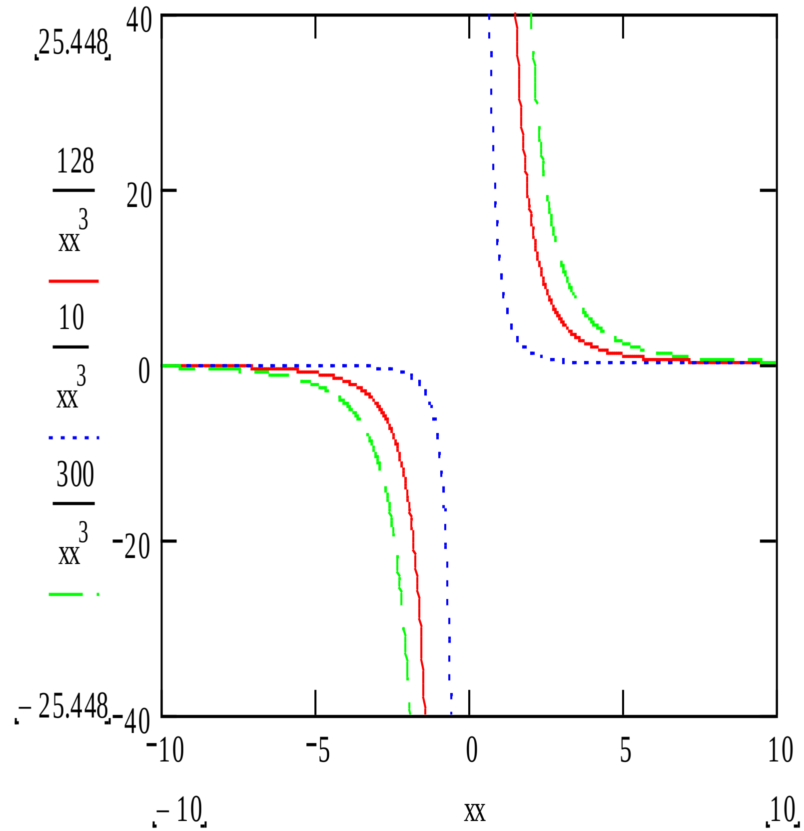
$$\frac{dy}{y} = -3 \cdot \frac{y}{x} \qquad \ln(y) = -3 \cdot \ln(x) + \ln(C)$$

$$y = e^{-3 \cdot \ln(x)} \cdot e^{\ln(C)} = \frac{C}{x^3}$$

At P $\quad x := 4 \qquad y := 2 \qquad C := 1$

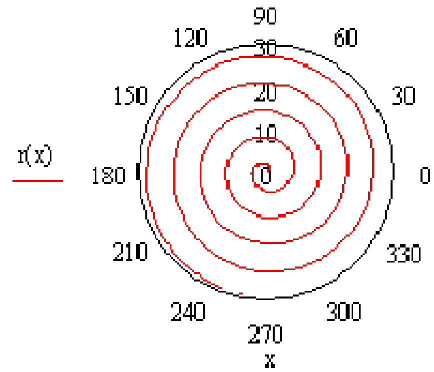
given

$$y = e^{-3 \cdot \ln(x)} \cdot e^{\ln(C)} \qquad C := \mathrm{Find}(C) \qquad C = 128$$



$$E = (400 \cdot y \cdot ax) + 400 \cdot x \cdot ay$$

$$A := \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$



$$\frac{dy}{dx} = \frac{Ey}{Ex} = \frac{x}{y}$$

$$x \cdot dx = y \cdot dx$$

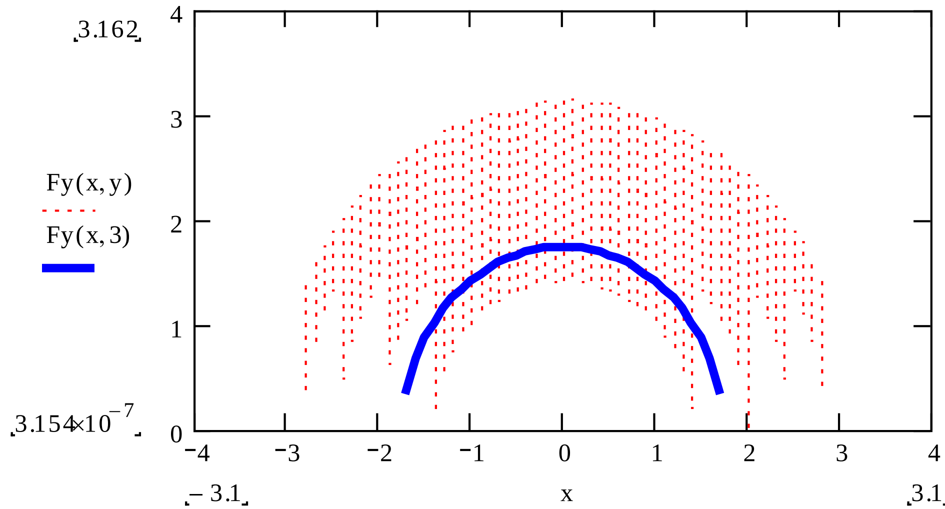
$$x^2 = y^2 + C$$

At A

$$C := 3$$

$$\frac{x^2}{3} - \frac{y^2}{3} = 1$$

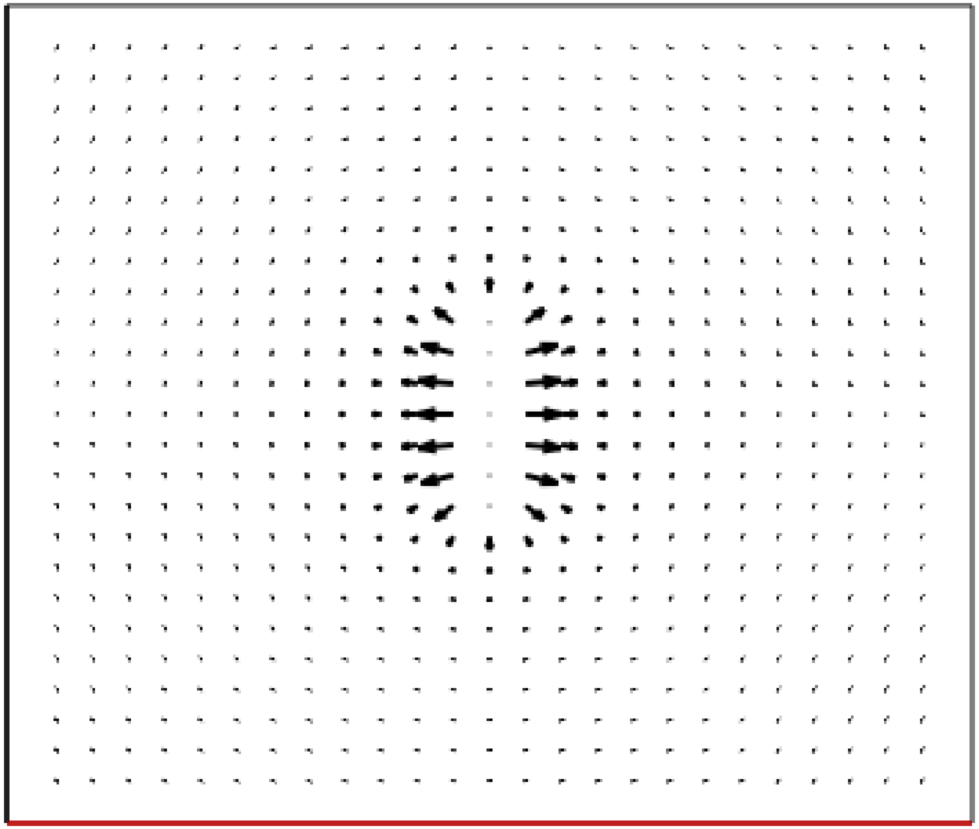
$$y(x) := \sqrt{3 - x^2}$$



Field of a Line Charge

MathCAD Example

Force field in the x = 0 plane.

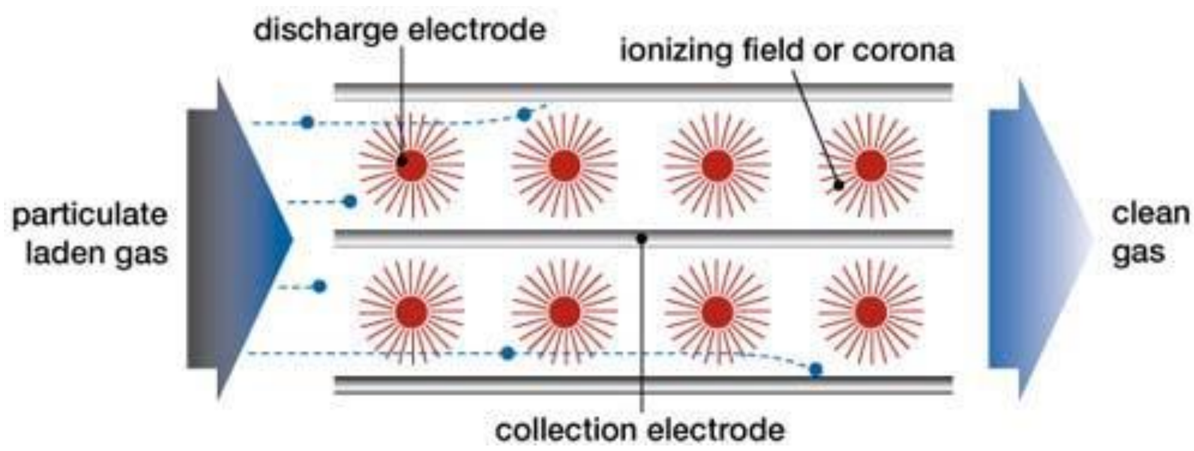


(f_y, f_z)

Electrostatic Precipitators for Power Plants

Many countries around the world, including our own, depend on coal and other fossil fuels to produce electricity. A natural result from the burning of fossil fuels, particularly coal, is the emission of flyash. Ash is mineral matter present in the fuel. For a pulverized coal unit, 60-80% of ash leaves with the flue gas. Historically, flyash emissions have received the greatest attention since they are easily seen leaving smokestacks.

Two emission control devices for flyash are the traditional fabric filters and the more recent electrostatic precipitators. The fabric filters are large baghouse filters having a high maintenance cost (the cloth bags have a life of 18 to 36 months, but can be temporarily cleaned by shaking or backflushing with air). These fabric filters are inherently large structures resulting in a large pressure drop, which reduces the plant efficiency. Electrostatic precipitators have collection efficiency of 99%, but do not work well for flyash with a high electrical resistively (as commonly results from combustion of low-sulfur coal).



Electrostatic precipitators are not only used in utility applications but also other industries (for other exhaust gas particles) such as cement (dust), pulp & paper (salt cake & lime dust), petrochemicals (sulfuric acid mist), and steel (dust & fumes).