Noise in Analog Modulation:

(A) **Receiver Model**: In formulating a receiver model for the study of noise in CW modulation systems, ~~we need to~~ following points must be considered:

- The model provides an adequate description of the form of receiver noise that is of common concern
- The model accounts for the inherent filtering and modulation characteristics of the system.
- The model is simple enough for a statistical analysis of the system to be possible

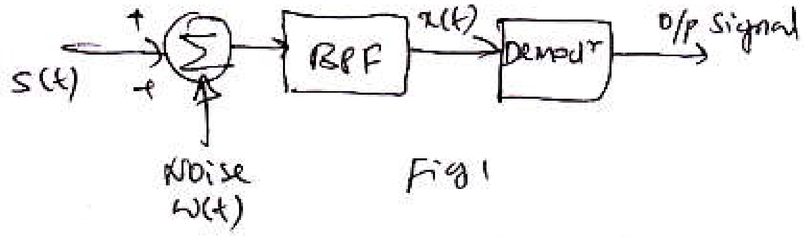


Fig 1

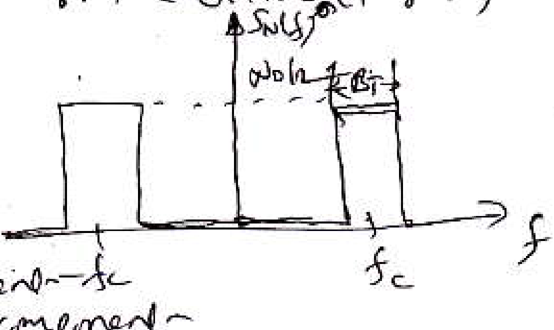
→ $s(t)$ denotes the incoming modulated signal and $w(t)$ denotes front-end receiver noise.

→ The receiver signal is therefore made up of the sum of $s(t)$ & $w(t)$

→ Bandpass filter is to pass the signal (desired) with enough bandwidth to pass the modulated signal

→ Power spectral density of noise is $N_0/2$ for both +ve & -ve frequencies i.e., N_0 is the average noise power per unit bandwidth measured at the front-end of the receiver.

→ Assuming mid freq of the filter is equal to carrier freq, we may model, the power spectral density $S_N(f)$ of the noise $n(t)$ resulting from the passage of the white noise $w(t)$ through the filter (fig. 2)



Representing filtered noise $n(t)$ as a narrowband noise represented in the canonical form

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin 2\pi f_c t$$

where → $n_I(t)$ is the in-phase noise component and $n_Q(t)$ is the quadrature noise component both measured w.r.t. the carrier wave $A_c \cos 2\pi f_c t$

∴ The filtered signal $x(t)$ available for demodulation is defined by
$$x(t) = s(t) + n(t)$$

$(SNR)_I \Rightarrow$ The ratio of the avg power of the modulated signal $s(t)$ to the avg power of the filtered noise $n(t)$.

⇒ A more useful measure of noise performance, however, is the output signal-to-noise ratio, $(SNR)_o$, defined as the ratio of the avg power of the demodulated message signal to the avg power of the noise, both measured at the receiver o/p.

⇒ $(SNR)_o$ provides (intuitively) measure for describing the fidelity with which the demodulation process in the receiver recovers the message signal from the modulated signal.

⇒ $(SNR)_o$ depends on:
- ~~*~~ The type of modulation used in the transmitter
- type of demodulation used in the ~~rx~~ receiver

For all systems to get meaningful analysis:
→ The modulated signal $s(t)$ transmitted by each system has the same avg power
→ The front end rxr noise $w(t)$ has the same avg power measured in the message bandwidth W.

Therefore, aditional reference we define the "channel signal-to-noise ratio, $(SNR)_C$ as the ratio of the avg power of the modulated signal to the avg power of noise in ~~the~~ the message bandwidth, both measured at the rxr i/p.

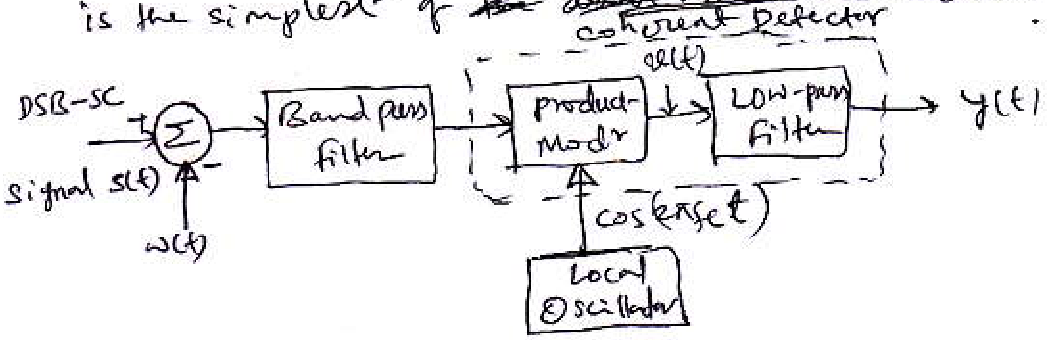
⇒ For the purpose of comparing different continuous-wave (CW) modulation systems, we normalize the receiver performance by dividing the o/p S/N ratio by the channel S/N ratio.
We thus define a figure of merit for the ~~rx as~~ receiver as
$$\text{Figure of merit} = \frac{(SNR)_o}{(SNR)_C}$$

Clearly, the higher the value of the figure of merit, the better will ~~be~~ the noise performance of receiver be.

⇒ The figure of merit may be '1', <1, or >1, depending on the type of modulation used.

Noise in DSB-SC Receivers:

⇒ Noise analysis of a DSB-SC receiver using coherent detection is the simplest of all cases. ~~the above mentioned cases.~~



⇒ Use of coherent detection requires multiplication of the filtered signal $x(t)$ by a locally generated sinusoidal wave $\cos(2\pi f_c t)$ & then low pass filtering the product.

⇒ The DSB-SC component of the filtered signal $x(t)$ is expressed as:

$s(t) = C A_c \cos(2\pi f_c t)\, m(t)$ where $A_c \cos(2\pi f_c t)$ is the sinusoidal carrier wave and $m(t)$ is the message signal.

$C \to$ system dependent scaling factor; included to get the same unit as that of white noise ~~w(t)~~ $n(t)$

⇒ Assuming $m(t)$ as the stationary process of zero mean, whose power spectral density $S_M(f)$ is limited to a maximum freq W (message BW).

⇒ The Avg. power P of the message signal is the total area under the curve of spectral density given by

$$P = \int_{-W}^{W} S_M(f)\, df.$$

⇒ The carrier wave is statistically independent of message signal i.e., the carrier should include a random phase that is uniformly distributed over 2π radians.

⇒ We know that Avg. power of DSB-SC is $C^2 A_c^2 P/2$; with a noise spectral density $N_0/2$, the avg noise power in the message bandwidth W is equal to $W N_0$. The channel S/N ratio of the DSB-SC modn is therefore

$$(SNR)_{C,DSBSC} = \frac{C^2 A_c P}{2 W N_0}$$

where the const C^2 in the numerator ensures that this ratio is dimensionless

⇒ Next step is to determine output S/N ratio of the system using the narrowband representation of the filtered noise

$n(t)$, the total signal at the coherent detector input may be expressed as:

$$x(t) = s(t) + n(t)$$

$$= C A_c \cos(2\pi f_c t)\, m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

where $n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature components of $n(t)$ with respect to the carrier. The o/p of the product modr component of the coherent detector is therefore

$$v(t) = x(t)\cdot\cos(2\pi f_c t)$$

$$= \tfrac{1}{2} C A_c m(t) + \tfrac{1}{2} n_I(t)$$

$$+ \tfrac{1}{2}\left[C A_c m(t) + n_I(t)\right]\cos(4\pi f_c t) - \tfrac{1}{2} A_c n_Q(t)\sin(4\pi f_c t)$$

The lowpass filter in the ~~independent~~ coherent detector removes the high freq components of $v(t)$, yielding a receiver o/p :

$$\boxed{y(t) = \tfrac{1}{2} C A_c m(t) + \tfrac{1}{2} n_I(t)}$$

Eqn $y(t)$ indicates: 1) The message signal $m(t)$ and inphase noise component $n_I(t)$ of the filtered noise $n(t)$ appear additively at the Rxr o/p.

2. The quadrature component $n_Q(t)$ of the noise $n(t)$ is completely rejected by the coherent detector.

⇒ From eqn $y(t)$ the noise component at the receiver o/p is $n_I(t)/2$, it follows that the avg power of the noise power at the receiver o/p is:

$$\left(\tfrac{1}{2}\right)^2 2WN_0 = \tfrac{1}{2} W N_0$$

⇒ The o/p S/N ratio for a DSB-SC receiver using coherent detection is therefore:

$$(SNR)_o = \frac{C^2 A_c^2 P/4}{W N_0/2} = \frac{C^2 A_c^2 P}{2 W N_0}$$

$$\Rightarrow \quad \frac{(SNR)_o}{(SNR)_c} = 1 \quad \text{for DSB-SC}$$

Note that factor C^2 is common to both o/p & channel S/N ratios.

Noise in AM Receivers: (using Envelop detector)

Type of modⁿ: AM

$$s(t) = A_c[1+k_a m(t)] \cos 2\pi f_c t. \rightarrow (1)$$

Step 1: determining the channel SNR, and then the step 2 (2) o/p SNR.

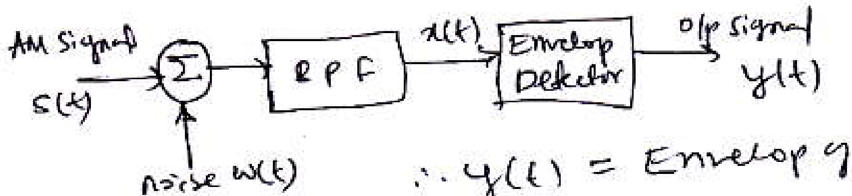
⇒ The average power of the carrier component in the AM signal s(t) is $A_c^2/2$. The Avg. power of the information-bearing component $A_c k_a m(t) \cos(2\pi f_c t)$ is $A_c^2 k_a^2 p/2$ where 'p' is the avg. power of the message signal m(t).

⇒ The avg. power of the full AM signal s(t) is therefore equal to $A_c^2(1+k_a^2 p)/2$. As for the DSB-SC system, the avg. power of noise in the message bandwidth is WN_0. The channel SNR for AM is therefore

$$(SNR)_{c,AM} = \frac{A_c^2(1+k_a^2 p)}{2WN_0}.$$

O/p SNR: To evaluate the o/p SNR, we first represent the filtered noise n(t) in terms of its in-phase and quadrature phase components. We may therefore define the filtered signal x(t) applied to the envelop detector in the receiver model as follows

$$x(t) = s(t) + n(t)$$
$$= [A_c + A_c k_a m(t) + n_I(t)] \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$$



$$\therefore y(t) = \text{Envelop of } x(t)$$
$$= \{[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t)\}^{1/2}$$

The signal y(t) defines the o/p of an ideal envelop detector. The phase of x(t) is of no interest because an ideal envelop detector is totally insensitive to variations in the phase of x(t)

⇒ The expression defining y(t) is somewhat complex and needs to be simplified in some manner in order to permit the derivation of insightful results

⇒ Assuming $A_c[1+k_a m(t)] >>$ white noise terms $n_I(t)$ and $n_Q(t)$ ⇒ $y(t) = A_c + A_c k_a m(t) + n_I(t)$

Accordingly, the o/p signal-to-noise ratio of an AM receiver using an envelop detector is approximately

$$(SNR)_{o,AM} = \frac{A_c^2 K_a^2 P}{2W N_0}$$

This eqn is valid if the following conditions are satisfied

1) The Avg. noise power is small compared to the average carrier power at the envelop detector input

2) The amplitude sensitivity K_a is adjusted for a percentage modn less than or equal to 100%.

$$\Rightarrow \left.\frac{(SNR)_o}{(SNR)_c}\right|_{AM} = \frac{K^2 P}{1+K_a^2 P} \quad \text{which is always} < 1$$

Example: Single tone modn: Consider the special case of a sinusoidal wave of frequency f_m and amplitude A_m as the modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

The corresponding AM ~~waves~~ wave is

$s(t) = A_c[1+\mu\cos(2\pi f_m t)]\cos 2\pi f_c t$ where $\mu = K_a A_m$ is the modulation factor or M.I.

The Avg. power of the modulating wave $m(t)$ is (assuming 1Ω resistor)

$$P = \frac{A_m^2}{2}$$

$$\therefore \left.\frac{(SNR)_o}{(SNR)_c}\right|_{AM} = \frac{\frac{1}{2}K_a^2 A_m}{1+\frac{1}{2}K_a^2 A_m^2} = \frac{\mu^2}{2+\mu^2}$$

When $\mu = 1$, (100% modn) we get figure of merit $\frac{1}{3}$. This means that other factors being equal, an AM System (using envelop detn) must transmit three times as much avg power as a DSB-SC (using coherent detn) in order to achieve same quality of noise performance.

Threshold Effect in AM:

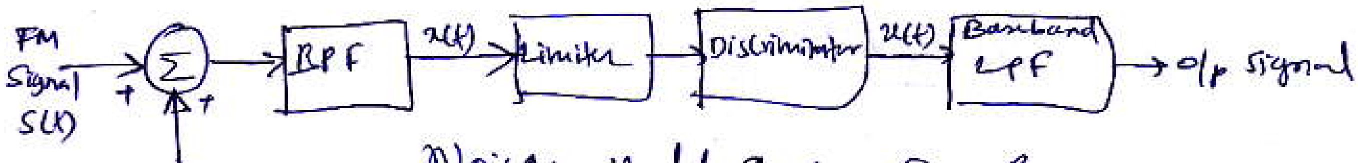
The loss of a message in an envelop detector that operates at a low carrier-to-noise ratio is referred to as the threshold effect,

⇒ By threshold we mean a value of the carrier-to-noise ratio below which the noise performance of a detector deteriorates much more rapidly than proportionately to the carrier-to-noise ratio.

⇒ It is important to recognize that every nonlinear detector (e.g. envelop detector) exhibits a threshold effect.

⇒ On the other hand, such an effect does not arise in a coherent-detector (linear)

Noise in FM receivers:



Noisy Model of an FM Rxr

Noise ($w(t)$) ⇒ white gaussian noise
Zero mean
Power Spect den. = $N_0/2$

(As before i.e., in AM)

The discriminator in the model consists of two components:

→ A slope detector with a purely imaginary transfer function that varies linearly with frequency. produces hybrid modulated wave (FM + AM)

→ An envelope detector that recovers the amplitude variation and thus reproduces the message signal.

⇒ The filtered noise $n(t)$ at the band-pass filter output is defined in terms of its in-phase and quadrature phase components by

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

Equivalently, $n(t)$ is expressed as:

$$n(t) = r(t)\cos[(2\pi f_c t + \psi(t)]$$

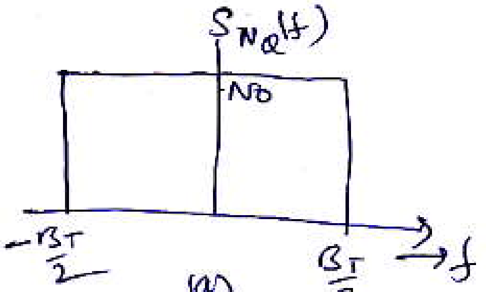
where the envelop is $r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2}$

& phase $\psi(t) = \tan^{-1}[n_Q(t)/n_I(t)]$

{psd → power spectral density}

The power spectral density $S_{N_0}(f)$ of the noise $n_0(t)$ appearing at the receiver o/p is defined by

$$S_{N_0}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2} & |f| \le W \\ 0, & \text{otherwise} \end{cases}$$



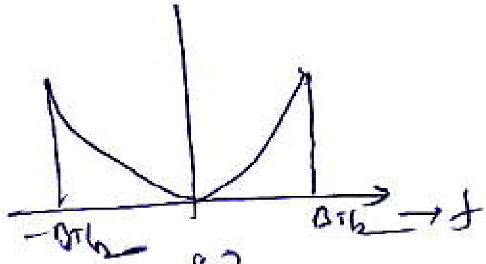
(a)

$\boxed{S_{N_d}(f) = \dfrac{f^2}{A_c^2} S_{N_Q}(f)}$

The avg. o/p noise power is determined by integrating the power spectral density $S_{N_0}(f)$ from $-W$ to W.

$\Rightarrow$ Avg. power of o/p noise $= \dfrac{N_0}{A_c^2}\displaystyle\int_{-W}^{W} f^2\,df$

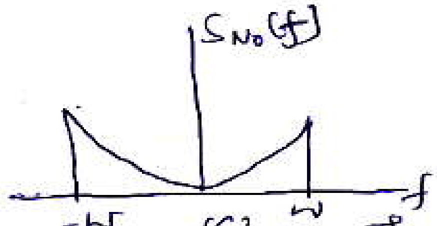
$$= \frac{2 N_0 W^3}{3 A_c^2}$$



(b)

Also, Avg. o/p signal power is $k_f^2 P$.

$$\therefore \quad (SNR)_{O,FM} = \frac{3 A_c^2 k_f^2 P}{2 N_0 W^3}$$



(c)

Noise analysis of FM Rxr a) psd of quadrature component $n_Q(t)$ of narrowband noise $n(t)$. (b) power spectral density of $n_d(t)$ at discriminator o/p (c) power spectral density of $n_0(t)$ at Rxr o/p.

The avg. power in the modulated signal $s(t)$ is $A_c^2/2$ and the avg noise power in the message BW is $W N_0$. Thus the channel signal-to-noise ratio is

$$(SNR)_{C,FM} = \frac{A_c^2}{2 W N_0}$$

$$\Rightarrow \left.\frac{(SNR)_O}{(SNR)_C}\right|_{FM} = \frac{3 k_f^2 P}{W^2}$$

Note that deviation ratio D (volts/Hz) is proportional to $k_f\sqrt{P}/W$. This follows that the figure of merit of a wide band FM system is a quadratic function of the deviation ratio.

Also, in WFM, transmission bandwidth B_T is approximately proportional to the deviation ratio D.

$\Rightarrow$ When carrier-to-noise is high, an increase in B_T provides increase in $(SNR)_0$ & hence increase in fig. of merit.

Example: Consider a single tone modulation with

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t)\right]$$

Therefore, we may write

$$2\pi K_f \int^t m(\tau)\, d\tau = \frac{\Delta f}{f_m}\sin(2\pi f_m t)$$

Differentiating both sides with respect to time & solving for $m(t)$

$$m(t) = \frac{\Delta f}{K_f}\cos(2\pi f_m t)$$

Hence, the avg. power of the message signal $m(t)$, developed across a 1-ohm load, is $\quad P = \dfrac{(\Delta f)^2}{2K_f^2}$

substituting this result into the formula for the o/p signal-to-noise ratio into $(SNR)_{o\,FM} = \dfrac{3A_c^2 K_f^2 P}{2N_0 W^3}$

$$= \frac{3A_c^2(\Delta f)^2}{4N_0 W^3}$$

$$= \frac{3A_c^2\beta^2}{4N_0 W}$$

where $\beta = \Delta f/W$ is the modulation index. Using $\left.\dfrac{(SNR)_o}{(SNR)_c}\right|_{FM} = \dfrac{3K_f^2 P}{W^2}$

$$= \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2 = \frac{3}{2}\beta^2$$

It is important to note that the modulation index $\beta = \Delta f/W$ is determined by the bandwidth W of the post detection low-pass filter & is not related to the sinusoidal message freq f_m

Comparison of AM & FM w.r.t. noise:

⇒ It is of particular interest to compare the noise performance of AM and FM systems. An insightful way of making this comparison is to consider the figure of merit of the two systems based on a sinusoidal modulating signal.

⇒ For an AM ~~signal~~ system operating with sinusoidal modulating signal and 100% modulation

$$\frac{(SNR)_o}{(SNR)_c} = \frac{1}{3}.$$

Comparing this figure of merit with the corresponding result for an FM system, we see that the use of freq modulation offers the possibility of improved noise performance over amplitude modulation when $\frac{3}{2}\beta^2 > \frac{1}{3}$ i.e, $\beta > \frac{\sqrt{2}}{3} = 0.471$

We may therefore consider $\beta = 0.5$ as defining roughly, the transition between narrowband FM & wideband FM. This statement based on noise considerations, further conforms a similar observations that was made ~~~~ while considering the bandwidth of FM waves

~~Capture Effect~~:

Capture Effect:- The inherent ability of an FM system to minimize the effects of ~~~~ unwanted signals (noise) also applies to interference by another frequency modulated signal whose freq. content is close to the carrier freq of the desired FM wave. However, interference suppression in an FM ~~wave~~ receiver works well only when the interference is weaker than the desired FM input. When the interference is the stronger one of the two, the receiver locks ~~~~ onto the stronger signal and thereby suppresses the desired FM input.

When they are of nearly equal strength, the receiver fluctuates back and forth between them. This phenomenon is known as the capture effect, which describes another distinctive characteristic of freq modulation.

FM Threshold effect:-

In FM receiver output signal to noise ratio ~~is~~ is valid only when (measured at the discriminator input) is high compared with unity. It is found experimentally that as the input noise power is increased so that the carrier-to-noise ratio is decreased, the FM receiver breaks.

At first, individual clicks are heard in the receiver output, and as the carrier-to-noise ratio decreases still further, clicks rapidly merge into a crackling or sputtering sound. Near the breaking point output signal to noise ratio $(SNR)_o$ begins to fail by predicting values of output signal-to-noise ratio larger than actual ones. This phenomenon is known as the threshold effect. The threshold is defined as the minimum carrier-to-noise ratio yielding an improvement that is not significantly deteriorated from the values predicted by the usual signal-to-noise formula assuming a small noise power.

To characterize threshold performance, carrier-to-noise ratio be defined by

$$\rho = \frac{A_c^2}{2B_T N_0}$$

As ρ is decreased, the avg number of clicks per unit time increases. When this number becomes appreciably large, the threshold is said to occur.

The output signal-to-noise ratio is calculated as follows:

1. The output signal is taken as the receiver o/p measured in the absence of noise.
2. The avg. output noise power is calculated when there is no signal present: i.e., the carrier is unmodulated, with no restriction imposed on the value of the carrier-to-noise ratio ρ.

We may conclude that threshold effects in FM receivers may be avoided in most practical cases of interest if the carrier-to-noise ratio 'ρ' is equal to or greater than 20 or, equivalently, 13 dB.

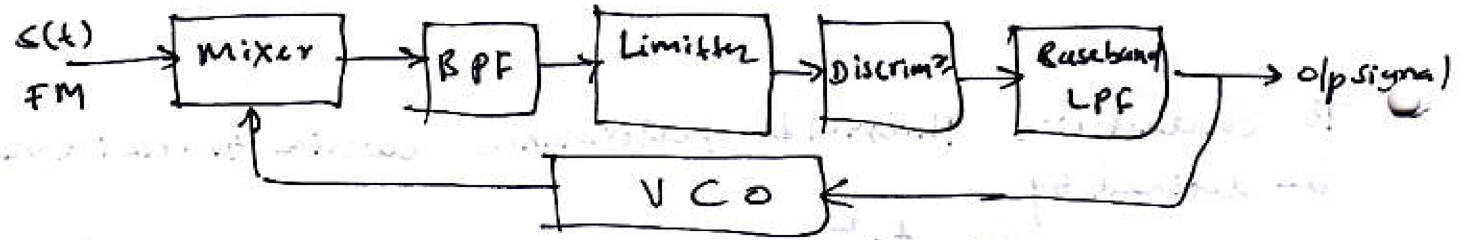
i.e., $$\frac{A_c^2}{2B_T N_0} \geq 20$$

or, equivalently, if the average transmitted power $A_c^2/2$ satisfies the condition: $A_c^2/2 \geq 20 B_T N_0$.

To use this formula, we may proceed as follows:

1. For a specified modulation index β and message band-width W, we determine transmission bandwidth of the FM wave B_T, using Carson's rule.
2. For a specified average noise power/unit BW, N_0 we use $\frac{A_c^2}{2} \geq 20 B_T N_0$ to determine the minimum value of the transmitted avg. power $A_c^2/2$ that is necessary to operate above threshold.

FM Threshold Reduction:



FMFB Demodulator Rxr.

In certain applications such as space communications using freq modulation, there is particular interest in reducing the noise threshold in an FM receiver so as to satisfactorily operate the receiver with the minimum signal power possible.

Threshold reduction in FM receivers may be achieved by using an FM demodulator with negative feedback (known as FMFB) or by using PLL. Such devices are called extended-threshold demodulators

⇒ In block diagram, Local oscillator is replaced by VCO, whose instantaneous freq is controlled by demodulated signal.