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ECE Dept.
Exam.
Internal Assessment
Odd Sem(2018-19)

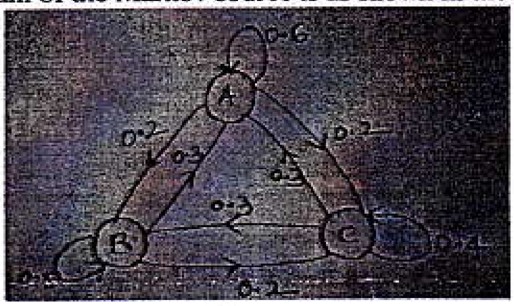
FIRST INTERNAL ASSESSMENT

Sem.: V Semester
Date: 11/09/2018

Sub: Information Theory and Coding
Time: 3.00PM to 4.00PM

Sub. Code: 15EC54
Max. Marks: 25

Note: Answer two full questions, draw sketches wherever necessary.

Q. No		Discription of Question	Marks	CO	RBT LEVEL
1	a	Derive an expression for the entropy of n^{th} extension of a zero memory source	6	CO304.1	L2
	b	A source emits one of the four probable messages m1, m2, m3, and m4 with probabilities 3/11, 2/11, 2/11, and 4/11 respectively. Find the entropy of the source.	7	CO304.1	L3
		OR			
2	a	State and prove the extermal property of entropy with an example	6	CO304.1	L2
	b	The state diagram of the Markov source is as shown in the Figure below:  i) Find the stationary distribution ii) Find the entropy of each state and hence the entropy of the source. iii) Find the entropy of the adjoint source and verify that $H(S)<H(\overline{S})$	7	CO304.1	L3
3	a	Obtain an expression for maximum entropy of a system	6	CO304.1	L2
	b	A black and white TV receiver consists of 625 lines of picture information. Assume that each line consists of 625 picture elements (pixels) and that each element can have 128 brightness levels. Picture are repeated at the rate of 25 frames/sec. Calculate the average rate of information conveyed by a TV set to a viewer.	6	CO304.1	L3
		OR			
4	a	Explain the significance of Markov model with state diagram.	4	CO304.1	L1
	b	A code is composed of dots and dashes. Assume a dash is 2 times as long as a dot and has 1/4 probability of occurance. Calculate: i) The information in a dot and a dash. ii) The entropy of a dot-dash code. iii) The average rate of information if a dot lasts for 10ms and the double time is allowed between symbols.	8	CO304.1	L3

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IA - I SCHEME OF EVALUATION

Sem : V	Subject : Information Theory & Coding	Sub Code : 15EC54	Date : 11/09/2018		
Q. No.	**Bit**	**Description**	**Marks**	**CO's**	**RBT LEVEL**
1	(a)	Let $S=\{S_1, S_2\}$ with $P=\{P_1, P_2\}$ so that $P_1+P_2=1$. Then its 2nd extension: $S^2=\{S_1S_1, S_1S_2, S_2S_1, S_2S_2\}$ with probabilities $P^2=\{P_1^2, P_1P_2, P_2P_1, P_2^2\}$ And, again $P_1^2+P_1P_2+P_2P_1+P_2^2=(P_1+P_2)^2=1$ Now $H(S)$ = Entropy of the basic binary source $= \sum_{i=1}^{2} P_i \log \frac{1}{P_i}$ $\therefore H(S) = P_1 \log\frac{1}{P_1} + P_2 \log\frac{1}{P_2}$ $\therefore H(S^2) = \sum_{j=1}^{4} P_j \log\frac{1}{P_j}$ $= P_1^2 \log_2\frac{1}{P_1^2} + P_1P_2 \log_2\frac{1}{P_1P_2} + P_1P_2\log_2\frac{1}{P_1P_2} + P_2^2\log_2\frac{1}{P_2^2}$ on simplification $= 2\left(P_1\log\frac{1}{P_1} + P_2\log\frac{1}{P_2}\right) = 2H(S)$ $\therefore H(S^2) = 2H(S)$ IIIly $H(S^3)=3H(S)$ ---- $H(S^n) = nH(S)$	06	CO304.1	L2
	(b)	$S=\{m_1, m_2, m_3, m_4\}$ & $P=\{\frac{3}{11}, \frac{2}{11}, \frac{2}{11}, \frac{4}{11}\}$ $H(S) = \frac{3}{11}\log_2\frac{1}{3/11} + 2\times\frac{2}{11}\log_2\frac{1}{2/11} + \frac{4}{11}\log_2 1/4/11$ $= \frac{3}{11}\log_2 3.67 + 0.3636\log_2 5.5 + 0.3636\log_2 2.75$ $= 0.511 + 0.8942 + 0.5306 = \underline{1.936}$	07	CO304.1	L3
2	(a)	<u>Extremal property</u>: Let us consider the source S with q symbols i.e., $S=\{S_1, S_2, S_3, \cdots S_q\}$ with probabilities $P=\{P_1, P_2, P_3, \cdots P_q\}$ Then $H(S) = \sum_{i=1}^{q} P_i \log_2 1/P_i$ & $\sum_{\forall i} P_i = 1$. The upper bound can be calculated by considering a quantity $\log_2 q - H(S)$	06	CO304.1	L2

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		$\therefore \log q - H(S) = \left[\sum_{i=1}^{q} P_i\right] \log_2 q - \sum_{i=1}^{q} P_i \log \frac{1}{P_i}$ & $\sum_{\forall i} P_i = 1$. $= \sum_{i=1}^{q} P_i \left[\log q - \log \frac{1}{P_i}\right]$ $= \sum_{i=1}^{q} P_i \log q P_i$ $= \sum_{i=1}^{q} P_i \frac{\log_e q P_i}{\log_e 2}$ $\therefore \log q - H(S) = \log_2 e \sum_{i=1}^{q} P_i \ln q P_i$ WKT, $\ln x \leq x-1$ $\Rightarrow -\ln x \geq 1-x$ or $\ln 1/x \geq 1-x$ Assuming $x = \frac{1}{qP_i}$ $\Rightarrow \ln q P_i \geq 1 - 1/qP_i$ Multiplying both sides by P_i & taking $\sum_{i=1}^{q}$ $\Rightarrow \sum_{i=1}^{q} P_i \ln q P_i \geq \sum_{i=1}^{q} P_i - \sum_{i=1}^{q} 1/q$ using total probability theorem RHS = 0 $\Rightarrow \log q - H(S) \geq 0$ $\Rightarrow H(S) \leq \log_2 q$ $\Rightarrow H(S)_{max} = \log_2 q$ And, lower bound is always zero//			
2	(b)	$P(A) = 0.6P(A) + 0.3P(B) + 0.3P(C)$ — (1) $P(B) = 0.2P(A) + 0.5P(B) + 0.3P(C)$ — (2) $P(C) = 0.2P(A) + 0.2P(B) + 0.4P(C)$ — (3) Solving 3 simultaneous equations $P(A) = 3/7 = 0.4285$ $P(B) = 9/28 = 0.321$ & $P(A)+P(B)+P(C) = 1$. $P(C) = 1/4 = 0.250$	07	C0304.1	L3

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Q2 (b) cont'd.

WKT, $H_i = \sum_{i=A}^{C} P_{ij} \log_2 \frac{1}{P_{ij}}$

For state A:

$$H_A = P_{AA} \log_2 \frac{1}{P_{AA}} + P_{AB} \log_2 \frac{1}{P_{AB}} + P_{AC} \log_2 \frac{1}{P_{AC}}$$

$$= 0.6 \log \frac{1}{0.6} + 0.2 \log \frac{1}{0.2} + 0.2 \log \frac{1}{0.2}$$

$$= 1.371 \text{ bits/symbol} \; //$$

For state B: $H_B = P_{BA} \log \frac{1}{P_{BA}} + P_{BB} \log \frac{1}{P_{BB}} + P_{BC} \log \frac{1}{P_{BC}}$

$$= 0.3 \log \frac{1}{0.3} + 0.5 \log \frac{1}{0.3} + 0.2 \log \frac{1}{0.2}$$

$$= 1.485 \text{ bits/symbol} \; //$$

For state C: $H_C = P_{CA} \log \frac{1}{P_{CA}} + P_{CB} \log \frac{1}{P_{CB}} + P_{CC} \log \frac{1}{P_{CC}}$

$$= 0.3 \log \frac{1}{0.3} + 0.4 \log \frac{1}{0.4} + 0.3 \log \frac{1}{0.3}$$

$$= 1.571 \text{ bits/symbol} \; //$$

$\Rightarrow H(S) = P(A)H(A) + P(B)H(B) + P(C)H(C)$

$$= \frac{3}{7} \times 1.371 + \frac{9}{28} \times 1.485 + \frac{1}{4} \times 1.571$$

$$= 1.45 \text{ bits/symbol} \; //$$

$$H(\bar{S}) = P(A) \log \frac{1}{P(A)} + P(B) \log \frac{1}{P(B)} + P(C) \log \frac{1}{P(C)}$$

$$= 3/7 \log 1/(3/7) + 9/28 \log 1/(9/28) + 1/4 \log 1/(1/4)$$

$$= 1.55 \text{ bits/symbol}$$

$\Longrightarrow H(S) < H(\bar{S})$

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Q. No.	**Bit**	**Description**		**Marks**	**CO's**	**RBT LEVEL**
3	(a)	Can be prooved using External property : 2(a) $H(S) = \sum_{i=1}^{q} P_i \log 1/P_i$ Entropy of a source $S = \{S_1, S_2, S_3 \cdots S_q\}$ & $P = \{P_1, P_2, P_3 \cdots P_q\}$ with q symbols let $\log q - H(S) = [\sum_{i=1}^{q} P_i] \log q - \sum_{i=1}^{q} P_i \log \frac{1}{P_i}$ $= \sum_{i=1}^{q} P_i \frac{\log_e qP_i}{\log_e 2}$ using log inequality $\ln x \leq x-1$ $\log q - H(S) \geq 0$ $\Rightarrow H(S) \leq \log_2 q$ The equality sign holds good when $P_i - 1/q = 0 \;\; \forall i$ i.e., $P_i = 1/q \; ; \forall i = 1, 2 \ldots q$. $\Rightarrow H(S)_{max} = \log_2 q$ bits/symbol		06	CO304.1	L2
3	(b)	Total no of pixels in one frame $625 \times 625 = 390625$ Since 128 brightness levels, total possible combinations become $(128)^{625 \times 625} = 128^{390625}$ also, all are equiprobable $\Rightarrow I = H(S)_{max} = \log_2 q = \log_2 128^{390625}$ $= 390625 \log_2 2^7 = 390625 \times 7 \Rightarrow 2734375$ 27.34×10^5 bits/frame		06	CO304.1	L3

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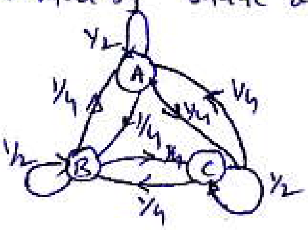
Q. No.	**Bit**	**Description**		**Marks**	**CO's**	**RBT LEVEL**
		Q3(b) cont'd ... WKT, $R_s = r_s \cdot H$. $r_s = 25$ frames/sec & $H = 27.34 \times 10^5$ bits/sym $\Rightarrow 25 \times 27.34 \times 10^5$ $= 68.25 \times 10^6$ bits/sec				
4	(a)	⇒ In zero memory sources, there will not be any intersymbol interference & in a long message, the symbols occur independently according to their $p(x_i)$ ⇒ However in real life cases messages depend on preceides q-symbols $(s_1, s_2 \ldots s_q)$ such a source is known as q^{th} order Markov source or Markoff source ⇒ Represented by state diagram e.g.,  A, B, C → states Self loops → next state = present state transition paths → state of next-stage.		04	CO304.1	L2
	(b)	Since only two symbols $P_{dot} + P_{dash} = 1$ & $P_{dash} = 1/4 \, P_{dot}$ $\Rightarrow P_{dot} + \frac{1}{4} P_{dot} = 1$ $= P_{dot} = 1/1.25 = 0.8$ // & $P_{dash} = 0.2$ //		08	CO304.1	L3

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H(S) cont'd..

i) $I_{dot} = \log_2 \frac{1}{0.8} = 0.3219$ bits

$I_{dash} = \log_2(1/0.2) = 2.32$ bits

ii) Entropy $H(S) = 0.8 \times 0.3219 + 0.2 \times 2.32$

$= 0.72$ bits/sym

iii) $P_{dot} = 4/5$ & $P_{dash} = 1/5$

For every 5 symbols sent, 1 is dash & 4 are dots.

dot period = 10 ms
dash " = 20 msec

. . . . —

4 → dots = 40 msec
1 → dash = 20 msec
Gap → 5 = 50 msec
×2 = 100 msec

$T_{tot} = 40 + 20 + 100 = 160$ msec

$\therefore r_s = 5$ symbols / 160 msec

$= \frac{5000}{160}$ symbols/sec

$= 31.25$ symbols/sec

$\therefore$ Information Rate $= R_I = r_s \times H(S)$

$= 31.25 \times 0.72$

$= 22.5$ bits/sec

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