Department of Engg. Mathematics

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Finite differences

Content

- Newton’s Forward Interpolation Formula
- Newton’s Backward Interpolation Formula
- Newton’s Divided Difference Method
- Lagrange interpolating polynomial
- Numerical integration

Newton's Forward Interpolation Formula

$$y_p = (1+\Delta)^p y_0 = \left\{1 + p\Delta + \frac{p(p-1)}{\lfloor 2}\Delta^2 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 + \ldots\right\} y_0$$

or

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 + \ldots + \frac{p(p-1)\ldots[p-(n-1)]}{\lfloor n}\Delta^n y_0 \quad \ldots\ldots\ldots(1)$$

(since if $y = f(x)$ is polynomial of nth degree then $\Delta^{n+1} y_0$ and other terms will be zero)

- Forward Difference Table

Finite Differences

Forward Difference Table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5	y_5					

Newton's Backward Interpolation Formula

Taking $p = \frac{x - x_n}{h}$, we get the interpolation formula as:

$$P(x_n+ph) = y_0 + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \Lambda + \frac{p(p+1)(p+2)\ldots(p+n-1)}{n!}\nabla^n y_n$$

Example

Estimate f(42) from the following data using newton backward interpolation.

x:	20	25	30	35	40	45
f(x):	354	332	291	260	231	204

Solution

The difference table is:

x	f	∇f	$\nabla^2 f$	$\nabla^3 f$	$\nabla^4 f$	$\nabla^5 f$
20	354					
		- 22				
25	332		- 19			
		- 41		29		
30	291		10		-37	
		- 31		- 8		45
35	260		2		**8**	
		- 29		**0**		
40	231		**2**			
		- 27				
45	**204**					

Solution

Newton backward formula is:

P(x) =

$$y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n +$$

$$\frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n$$

$$P(42) = 204 + (-0.6)(-27) + \frac{(-0.6)(0.4)}{2}\times 2 + \frac{(-0.6)(0.40(1.4)}{6}\times 0 +$$

$$\frac{(-0.6)(0.4)(1.4)(2.4)}{24}\times 8 + \frac{(-0.6)(0.4)(1.4)(2.4)(3.4)}{120}\times 45 = 219.1430$$

Thus, f(42) = 219.143

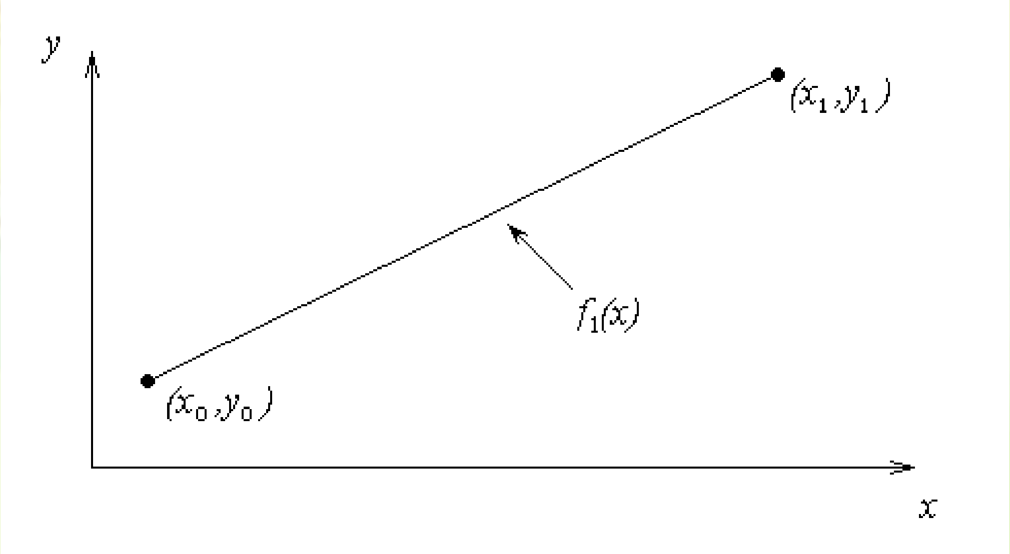
Newton's Divided Difference Method

Linear interpolation: Given $(x_0, y_0), (x_1, y_1),$ pass a linear interpolant through the data

$$f_1(x) = b_0 + b_1(x - x_0)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



General Form

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f[x_0] = f(x_0)$$

$$b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0} = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Rewriting

$$f_2(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1)$$

$f(x)$	First divided differences	Second divided differences	Third divided differences
$f[x_0]$			
	$f[x_0,x_1]=\dfrac{f[x_1]-f[x_0]}{x_1-x_0}$		
$f[x_1]$		$f[x_0,x_1,x_2]=\dfrac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$	
	$f[x_1,x_2]=\dfrac{f[x_2]-f[x_1]}{x_2-x_1}$		$f[x_0,x_1,x_2,x_3]=\dfrac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0}$
$f[x_2]$		$f[x_1,x_2,x_3]=\dfrac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$	
	$f[x_2,x_3]=\dfrac{f[x_3]-f[x_2]}{x_3-x_2}$		$f[x_1,x_2,x_3,x_4]=\dfrac{f[x_2,x_3,x_4]-f[x_1,x_2,x_3]}{x_4-x_1}$
$f[x_3]$		$f[x_2,x_3,x_4]=\dfrac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2}$	
	$f[x_3,x_4]=\dfrac{f[x_4]-f[x_3]}{x_4-x_3}$		$f[x_2,x_3,x_4,x_5]=\dfrac{f[x_3,x_4,x_5]-f[x_2,x_3,x_4]}{x_5-x_2}$
$f[x_4]$		$f[x_3,x_4,x_5]=\dfrac{f[x_4,x_5]-f[x_3,x_4]}{x_5-x_3}$	
	$f[x_4,x_5]=\dfrac{f[x_5]-f[x_4]}{x_5-x_4}$		
$f[x_5]$			

The Lagrange interpolating polynomial passing through three given points; (x_0, y_0), (x_1, y_1) and (x_2, y_2) is:

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

Numerical integration

Integration: The process of measuring the area under a curve.

$$I = \int_a^b f(x)dx$$

Where:

f(x) is the integrand

a= lower limit of integration

b= upper limit of integration

- **Simpson's 1/3rd Rule:**

Simpson's 1/3rd rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.

$$I = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + - - - - -) + 2(y_2 + y_4 + y_6 + - - - - -)]$$

- **Simpson's 3/8th Rule:**

$$I = \frac{3h}{8}[(y_0 + y_n) + 2(y_1 + y_2 + y_4 + - - - - -) + 3(y_3 + y_6 + y_9 + - - - - -)]$$

- **Weddle's Rule:**

$$I = \frac{3h}{10}[y_0 + 5y_1 + y_2 + 6\,y_3 + y_4 + 5y_5 + y_6]$$

Queries …?