Department of Engg. Mathematics

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Statistical Methods

Content

- Correlation
- Karl Pearson's Coefficient of Correlation
- Regression Equation of Line
- Curve fitting & Optimization
- Solution of Algebraic and Transcendental Equations
- Method of false position or Regula-Falsi Method
- Newton-Raphson Method

Correlation

- The degree of relationship between the variables under consideration is measure through the correlation analysis.
- The measure of correlation called the correlation coefficient .
- The degree of relationship is expressed by coefficient which range from correlation **(-1 ≤ r ≥ +1)**
- The direction of change is indicated by a sign.
- The correlation analysis enable us to have an idea about the degree & direction of the relationship between the two variables under study.

Type of Correlation

- **Positive Correlation:** The correlation is said to be positive correlation if the values of two variables changing with same direction.

  Ex. Pub. Exp. & sales, Height & weight.

- **Negative Correlation:** The correlation is said to be negative correlation when the values of variables change with opposite direction.

  Ex. Price & qty. demanded.

Karl Pearson's Coefficient of Correlation

- Pearson's 'r' is the most common correlation coefficient.
- Karl Pearson's Coefficient of Correlation denoted by- 'r' The coefficient of correlation 'r' measure the degree of linear relationship between two variables say x & y.
- Karl Pearson's Coefficient of Correlation denoted by- r

  $-1 \leq r \geq +1$
- Degree of Correlation is expressed by a value of Coefficient
- Direction of change is Indicated by sign (- ve) or (+ ve)
- When deviation taken from actual mean:

  $$r(x, y) = \Sigma xy / \sqrt{\Sigma x^2 \Sigma y^2}$$

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Regression Equation of Line

- **Regression Equation of y on x:**

$$Y - Y = b_{yx} (X - X)$$

$$b_{yx} = \sum xy / \sum x^2$$

$$b_{yx} = r (\sigma y / \sigma x)$$

- **Regression Equation of x on y:**

$$X - X = b_{xy} (Y - Y)$$

$$b_{xy} = \sum xy / \sum y^2$$

$$b_{xy} = r (\sigma x / \sigma y)$$

Curve fitting & Optimization

- **Regression Equation of y on x**

  $Y = a + bx$

  In order to obtain the values of ‘a’ & ‘b’

  $\sum y = na + b\sum x$

  $\sum xy = a\sum x + b\sum x^2$

- **Regression Equation of x on y**

  $X = a + by$

  In order to obtain the values of ‘c’ & ‘d’

  $\sum x = na + b\sum y$

  $\sum xy = a\sum y + b\sum y^2$

- Graphs of the form $y = ae^{nx}$

Taking natural logarithms of both sides of the equation:

yields: $$\ln y = \ln a + nx$$

If data is collected for the *x* and *y* values then the *y* values must be converted to *Y* values where:

So that: $$Y = \ln y$$

$Y = \ln a + nx$: a straight line gradient *n*, vertical intercept $\ln a$

Curve Fitting

Curve of the form $y = ax + b$

The normal equations are

$$\sum y = a\sum x + nb$$

$$\sum xy = a\sum x^2 + b\sum x$$

Curve of the form $y = ax^2 + bx + c$

The normal equations are

$$\sum y = a\sum x^2 + b\sum x + nc$$

$$\sum xy = a\sum x^3 + b\sum x^2 + c\sum x$$

$$\sum x^2 y = a\sum x^4 + b\sum x^3 + c\sum x^2$$

Numerical Methods

- Limitations of analytical methods led to the evolution of Numerical methods. Numerical
- Methods often are repetitive in nature i.e., these consist of the repeated execution of the same procedure where at each step the result of the proceeding step is used. This process known as iterative process is continued until a desired degree of accuracy of the result is obtained.

Solution of Algebraic and Transcendental Equations

The equation f(x) = 0 said to be purely algebraic if f(x) is purely a polynomial in x.

If f(x) contains some other functions like Trigonometric, Logarithmic, exponential etc. then f(x) = 0 is called a Transcendental equation.

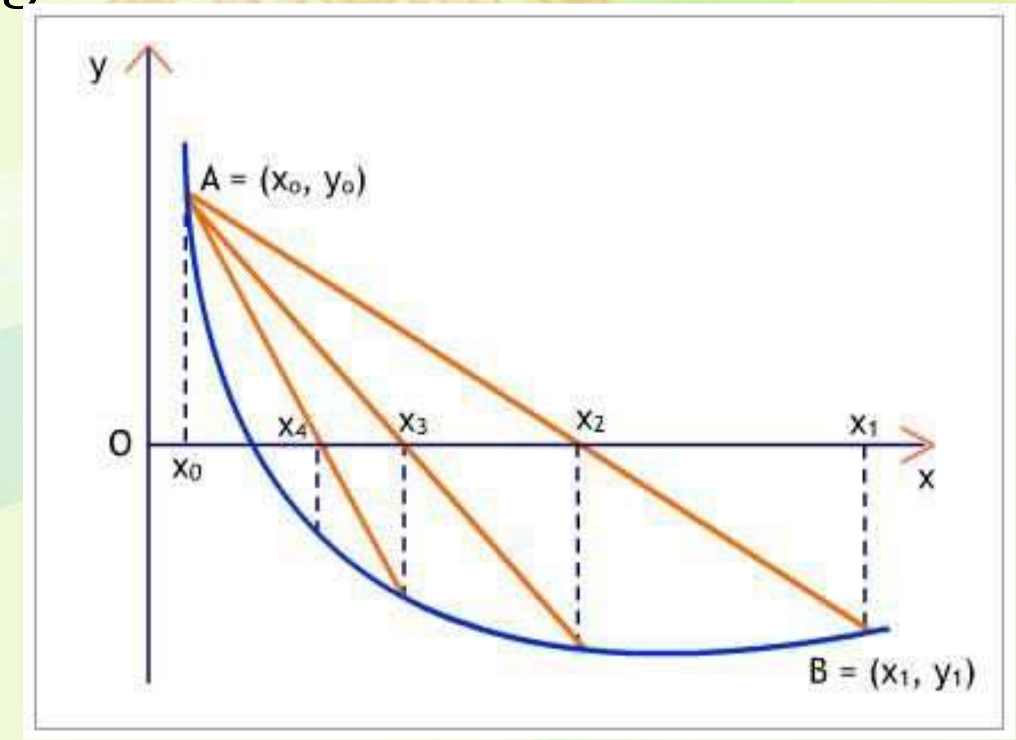
Ex: (1) $x^4 - 7x^3 + 3x + 5 = 0$ is algebraic

(2) $e^x - x \tan x = 0$ is transcendental

Method of false position or Regula-Falsi Method:

This is a method of finding a real root of an equation $f(x) = 0$ and is slightly an improvisation of the bisection method.

Let x_0 and x_1 be two points such that $f(x_0)$ and $f(x_1)$ are opposite in sign.



Let $f(x_0) > 0$ and $f(x_1) < 0$

$\therefore$ The graph of $y = f(x)$ crosses the x-axis between $x_0 = a$ and $x_1 = b$

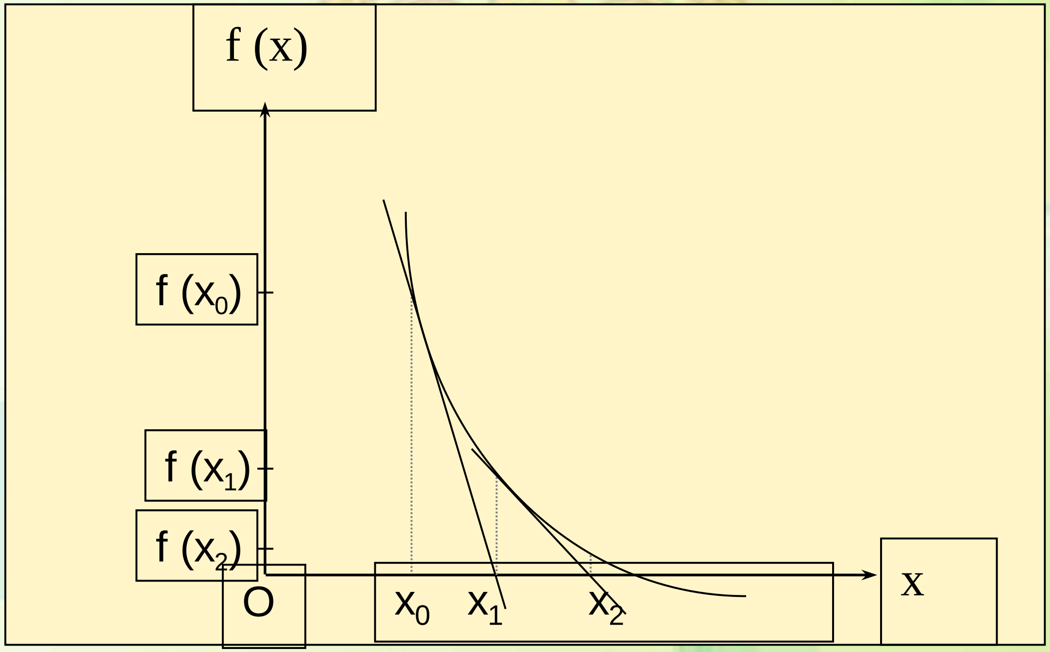
$\therefore$ Root of $f(x) = 0$ lies between $x_0 = a$ and $x_1 = b$

If f(a) and f(b) are opposite in sign then First approximation

$$y(x) = \frac{af(b) - b\,f(a)}{f(b) - f(a)}$$

Newton-Raphson Method

Tangent line of $f\ (x)$ is $f\ '(x)$. This is the basic of Newton-Raphson Method described below.



Gradient of the tangent line of $f(x)$ at $(x_0, f(x_0))$ is given by $f'(x_0)$. If we continue the tangent line at $(x_0, f(x_0))$ to $(x_1,0)$, then the gradient of this tangent line can be determined by

$$\frac{0 - f(x_0)}{x_1 - x_0}.$$

Equalizing these two values, we have

$$f'(x_0) = \frac{0 - f(x_0)}{x_1 - x_0}$$

or $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

- This means that from the initial value x_0, using formula (4), we can compute x_1 which is close to the solution of $f(x) = 0$.
- Using the same formula, we can compute x_2 which is a better approximation to the solution compare to x_1.
- We can also compute x_3 , x_4 , … and so on.
- We hope that our iteration converges to the solution of $f(x) = 0$.

In general, Newton-Raphson method can be formulated as

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.}$$

Queries …?