

Ohm's Law

21.8.18

This law gives relationship betⁿ potential diff (V), ct (I) & resistance (R) of a d.c. ckt. In 1827 Dr Ohm has invented this relation, which is called as Ohm's Law.

"The ct flowing through an elect. ckt is directly proportional to p.d. a/c the ckt & inversely proportional to resistance of the ckt".

Mathematically, $I \propto \frac{V}{R}$, $I = \frac{V}{R}$



Ohm's law can be applied either to the entire ckt or to the part of the ckt.

Limitations —

- 1) It is not applicable to non linear devices such as diodes, Zener diode, vg regulators etc
- 2) It is not applicable to non-metallic conductors i.e. semiconductors such as SiC.

Series & Parallel ckts

Characteristics of each,

Comparison

Ex ① Resistance of two wires is 25Ω when connected in series & 6Ω when connected in parallel. Calculate the resistance of two wires.

Solⁿ Let R_1 & R_2 be the two unknown resistances

Then, when in series $R_1 + R_2 = 25$ — (1) or $R_2 = 25 - R_1$

& when in parallel $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6}$ or $G = \frac{R_1 R_2}{R_1 + R_2}$ — (2)

Substituting the value of R_2 in eqⁿ 2 R_1 , we have

$$G = \frac{R_1 \times (25 - R_1)}{R_1 + 25 - R_1} \Rightarrow R_1^2 - 25R_1 + 150 = 0$$

$$\text{or } (R_1 - 15)(R_1 - 10) = 0$$

$$\therefore \text{if } R_1 = 15\Omega, \text{ then } R_2 = 10\Omega \text{ or } R_1 = 10\Omega, R_2 = 15\Omega$$

Ex ② The equivalent resistance of four resistors joined in parallel is 20Ω . The cts flowing through them are 0.6 , 0.3 , 0.2 & 0.1 A. Find the value of each resistor when ~~20V~~ is applied to the ckt

Solⁿ Total current in the ckt is

$$I_1 = 0.6 + 0.3 + 0.2 + 0.1$$

$$= 1.2 \text{ A}$$

Voltage applied a/c the resistors is

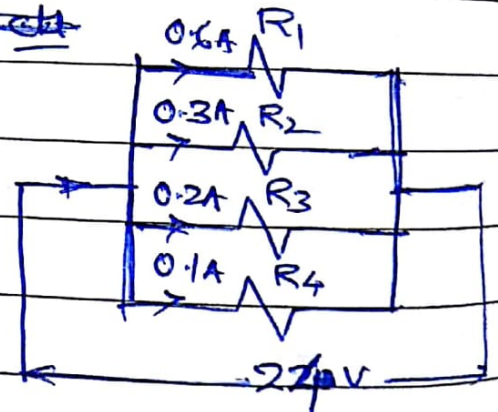
$$= 20 \times 1.2 = 24 \text{ V}$$

$$\therefore R_1 = 24 / 0.6 = 40\Omega$$

$$R_2 = 24 / 0.3 = 80\Omega$$

$$R_3 = 24 / 0.2 = 120\Omega$$

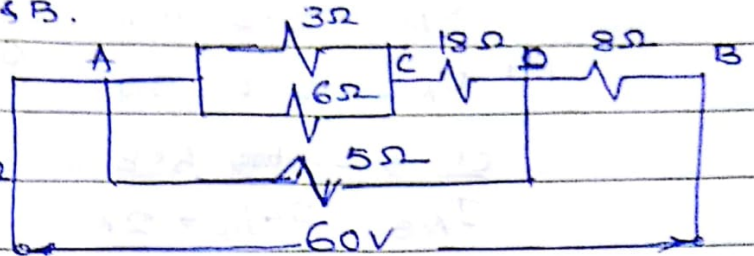
$$R_4 = 24 / 0.1 = 240\Omega$$



Ex ③ Calculate the effective resistance of the combination of resistances shown & the voltage drop a/c each resistance when a p.d. of 60V is applied between points A & B.

Solⁿ: Resistance betⁿ A & C

$$R_{AC} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$$



Resistance betⁿ A & D

$$R_{AD} = 2 + 18 = 20\Omega$$

$$R_{AD} = \frac{20 \times 5}{20+5} = \frac{100}{25} = 4\Omega$$

∴ Resistance betⁿ A & B

$$R_{AB} = 4 + 8 = 12\Omega$$

$$\text{Now Total ckt ct} = I = \frac{60}{12} = 5A$$

$$\text{ct through } 5\Omega \text{ } I_{5\Omega} = 5 \times \frac{20}{25} = 4A$$

$$\therefore \text{ct through AD Branch} = 5 - 4 = 1A$$

$$\text{Now ct through } 3\Omega = I_{3\Omega} = 1 \times \frac{6}{9} = 0.67A$$

$$\therefore \text{ct through } 6\Omega = I_{6\Omega} = 0.33A$$

$$\text{Now } \underline{v_g} \text{ a/c } 3\Omega = 3 \times 0.67 = 2V = \underline{v_g} \text{ a/c } 6\Omega \text{ resistor}$$

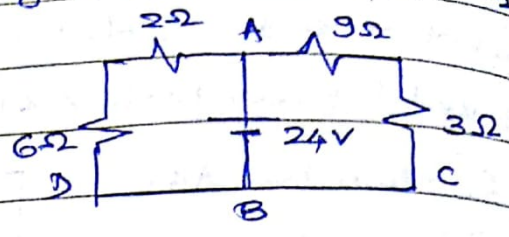
$$\underline{v_g} \text{ a/c } 18\Omega \text{ } V_{18} = 18 \times 1 = 18V$$

$$\underline{v_g} \text{ a/c } 5\Omega \text{ } V_5 = 5 \times 4 = 20V$$

$$\underline{v_g} \text{ a/c } 8\Omega \text{ } V_8 = 8 \times 5 = 40V$$

Ex 4 In the ckt shown, find the vg a/c ct in each resistor

Soln:
 9Ω & 3Ω are in series
 $\therefore R_{ACB} = 9 + 3 = 12\Omega$
 $\text{iff } R_{ADB} = 2 + 6 = 8\Omega$



\therefore ct in section ACB is
 $I_{ACB} = 24/12 = 2A$

& ct in section ADB is
 $I_{ADB} = 24/8 = 3A$

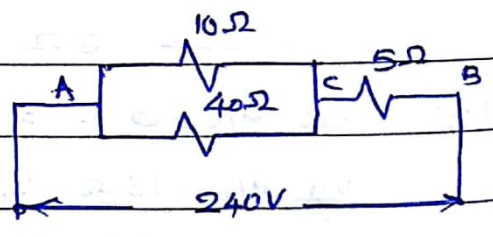
\therefore vg a/c 9Ω $V_3 = 9 \times 2 = 18V$ $V_2 = 2 \times 3 = 6V$
 $\text{iff } V_3 = 3 \times 2 = 6V$ $V_6 = 6 \times 3 = 18V$

Assignments:

1) Two resistances of 10Ω & 40Ω respectively are connected in parallel. A third resistance of 5Ω is connected in series with the combination & a dc supply of $240V$ is applied to the ends of the complete ckt. Calculate (i) ct in each resistance, (ii) what power would be spent in a fourth resistor of 20Ω connected in parallel with the 5Ω resistance.

Soln: $R_{AC} = \frac{10 \times 40}{50} = 8\Omega$

$R_{AB} = 8 + 5 = 13\Omega$



Total ct in the ckt $I = \frac{240}{13} = 18.46A$

ct in 10Ω resistor $I_{10} = 18.46 \times \frac{40}{50} = 14.77A$

\therefore ct in 40Ω - ? - $I_{40} = 18.46 - 14.77 = 3.7A$

- ? - 5Ω - ? - $I_5 = 18.46A$

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$$R_{AC} = 8 \Omega$$

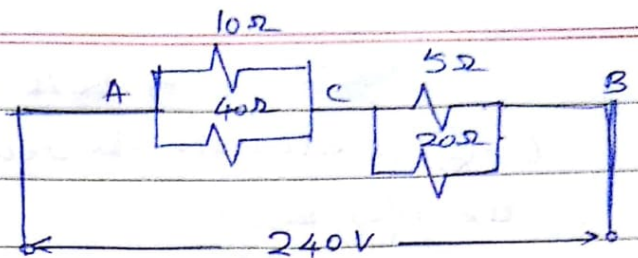
$$R_{CB} = \frac{5 \times 20}{25} = 4 \Omega$$

$$\therefore R_{AB} = 12 \Omega$$

$$I_T = \frac{240}{12} = 20 \text{ A}$$

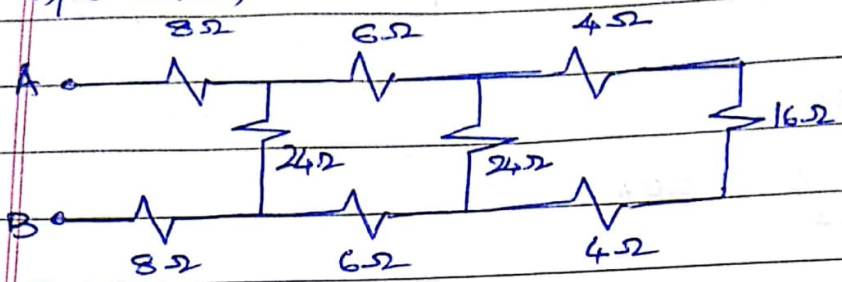
$$I_{10} = \frac{4}{20} \times \frac{40}{50} = 16 \text{ A} \quad \therefore I_{40} = 4 \text{ A}$$

$$I_{20} = 20 \times \frac{20}{25} = 16 \text{ A} \quad \therefore P = I^2 R = 16 \times 20 = \underline{320 \text{ W}}$$



Problems on KVL :

Ex 6) Calculate the equivalent resistance a/c the supply terminals in the n/w shown



Soln : Resistances 4Ω , 16Ω & 4Ω are in series \therefore

$$\therefore R_{1eq} = 4 + 16 + 4 = 24\Omega$$

R_{1eq} is in parallel with 24Ω resistor

$$\therefore R_{2eq} = \frac{24 \times 24}{48} = 12\Omega$$

Now R_{2eq} , 6Ω & 6Ω are in series

$$\therefore R_{3eq} = 6 + 12 + 6 = 24\Omega$$

R_{3eq} & 24Ω are in parallel

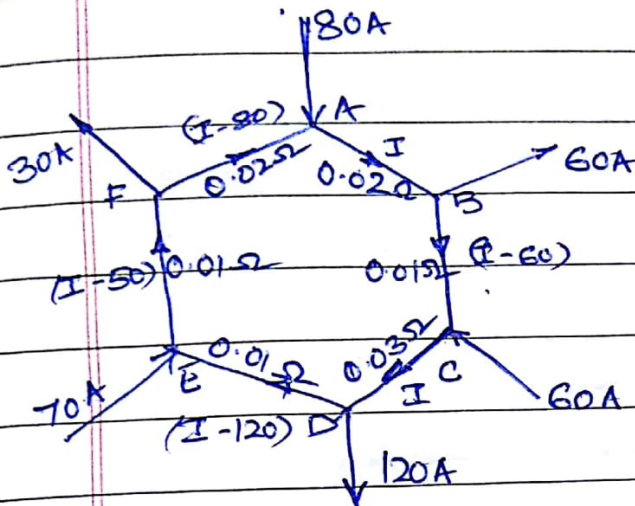
$$\therefore R_{4eq} = \frac{24 \times 24}{48} = 12\Omega$$

Now R_{4eq} , 8Ω & 8Ω are in series

$$\therefore R_{eq} = 8 + 12 + 8 = 28\Omega$$

Problems on KVL:

Ex ① Find the ct in all the branches of the g/w shown



Let ct through AB be I Amps.

Applying KCL at various nodes,
we can calculate branch ct

Applying KVL to the loop ABCDEFA

$$\begin{aligned}
 -I \times 0.02 - (I-60) \times 0.01 - I \times 0.03 - (I-120) \times 0.01 - (I-50) \times 0.01 - (I-30) \times 0.02 &= 0 \\
 -0.02I - 0.01I + 0.6 - 0.03I - 0.01I + 1.2 - 0.01I + 0.5 &= 0 \\
 -0.02I + 1.6 &= 0
 \end{aligned}$$

$$0.1I = 3.9$$

$$\therefore I = 39 \text{ A}$$

Now various branch cts are

$$AB \quad 39 \text{ A}$$

$$BC \quad -21 \text{ A}$$

$$CD \quad 39 \text{ A}$$

$$DE \quad -81 \text{ A}$$

$$EF \quad -11 \text{ A}$$

$$FA \quad -41 \text{ A}$$

Power & Energy -

Power (P) is the rate of doing work, or work done per unit time

$$P = \frac{VIt}{t} = V \times I \text{ Watts} = \frac{V^2}{R} \text{ or } I^2 R.$$

One watt may be defined as the rate of doing one joule / sec

Practical units of Power KW = 10^3 W or MW = 10^6 W.

Energy -

Total work done over a time is energy

$$E = P \times t = V \times I \times t \text{ W-sec}$$

Practical unit is kWh.

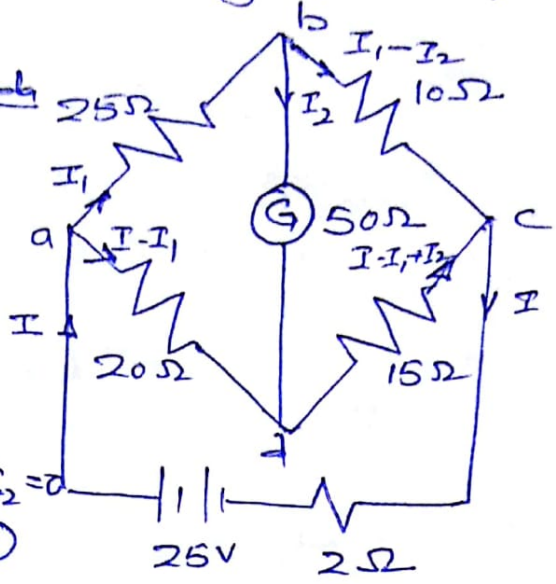
$$1 \text{ Wh} = 1 \text{ J/s} \times 3600 \text{ s} = 3600 \text{ J}$$

$$1 \text{ kWh} = 3600 \times 1000 = 36 \times 10^5 \text{ J}$$

Electricity bill is based on the no of kWh consumed.

Ex: Using Kirchhoff's laws, find the ct flowing through the galvanometer G in the Wheatstone bridge w/o star

Solⁿ Applying KVL to loop abcd



$$-25I_1 - 50I_2 + 20(I - I_1) = 0$$

$$-45I_1 - 50I_2 + 20I = 0 \quad \text{--- (1)}$$

bcdb

$$-10(I_1 - I_2) + 15(I - I_1 + I_2) + 50I_2 = 0$$

$$-25I_1 + 75I_2 + 15I = 0 \quad \text{--- (2)}$$

adca

$$-20(I - I_1) - 15(I - I_1 + I_2) - 2I + 25 = 0$$

$$+35I_1 - 15I_2 - 37I + 25 = 0 \quad \text{--- (3)}$$

$$\text{or } 35I_1 - 15I_2 - 37I = -25 \quad \text{--- (3)}$$

Applying Cramer's rule

$$D = \begin{vmatrix} -45 & -50 & 20 \\ -25 & 75 & 15 \\ 35 & -15 & -37 \end{vmatrix} = \frac{39750}{14750}$$

Now we want ct through galvanometer i.e. I_2 , so calculate D_2 only as I_2 is 2nd variable in the eq^s.

$$D_2 = \begin{vmatrix} -45 & 0 & 20 \\ -25 & 0 & 15 \\ 35 & -25 & -37 \end{vmatrix} = \frac{-4375}{14750}$$

$$\therefore I_2 = \frac{D_2}{D} = \frac{-0.0487}{1} = -0.0487A$$

Ex: A Wheatstone bridge ABCD is arranged as follows:

Resistances betⁿ AB, BC, CD, DA & BD are 10, 2, 8, 4 & 5 Ω resp. A 100V supply is connected betⁿ terminals A & C.

Det ct in branches AB & AD of the ckt & total ct taken from the supply.

Solⁿ Applying KVL to

ABDA

$$-10I_1 - 5I_2 + 4(I - I_1) = 0$$

$$-14I_1 - 5I_2 + 4I = 0 \quad \text{--- (1)}$$

BCDB

$$-2(I_1 - I_2) + 8(I - I_1 + I_2) + 5I_2 = 0$$

$$-10I_1 + 15I_2 + 8I = 0 \quad \text{--- (2)}$$

ADCA

$$-4(I - I_1) - 8(I - I_1 + I_2) + 100 = 0$$

$$12I_1 - 8I_2 - 12I = -100 \quad \text{--- (3)}$$

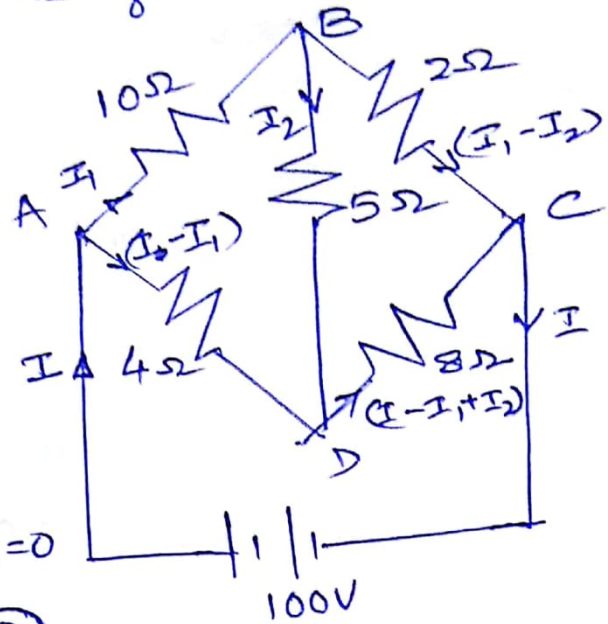
$$D = \begin{vmatrix} -14 & -5 & 4 \\ -10 & 15 & 8 \\ 12 & -8 & -12 \end{vmatrix} = 1624 + 120 - 400 = 1344$$

$$D_1 = \begin{vmatrix} 0 & -5 & 4 \\ 0 & 15 & 8 \\ -100 & -8 & -12 \end{vmatrix} = 4000 + 6000 = 10,000$$

$$I_1 = \frac{D_1}{D} = \frac{10000}{1344} = 7.44 \text{ A}$$

$$D_2 = \begin{vmatrix} -14 & 0 & 4 \\ -10 & 0 & 8 \\ 12 & -100 & -12 \end{vmatrix} = -1200 + 4000 = 2800$$

$$\therefore I_2 = D_2/D = \frac{2800}{1344} = 2.08 \text{ A}$$



Total ct

$$D_3 = \begin{vmatrix} -14 & -5 & 0 \\ -10 & 15 & 0 \\ 12 & -8 & -1.00 \end{vmatrix} = 21,000 + \cancel{25,000} \\ = \cancel{20,500} + 26,000 \\ \therefore I = \frac{D_3}{D} = \cancel{15.08A} 19.34A$$

Branch	Current
AB	$I_1 = 7.44A$
BC	$I_1 - I_2 = 12.79A$
CD	$I - I_1 + I_2 = 6.55A$
DA	$I - I_1 = 11.9A$
CA	$I = 19.34A$
BD	$I_2 = 5.35A$

Assgn

① A Wheatstone bridge is arranged as follows: Resistances between AB, BC, CD, DA & BD are 10, 20, 15, 5 & 40Ω resp. A 20V battery of negligible ~~battery~~ internal resistance is connected betⁿ A & C. Determine ct in each resistor.

Solⁿ Applying KVL to

ABDA

$$-10I_1 - 40I_2 + 5(I - I_1) = 0$$

$$-15I_1 - 40I_2 + 5I = 0 \quad \text{--- (1)}$$

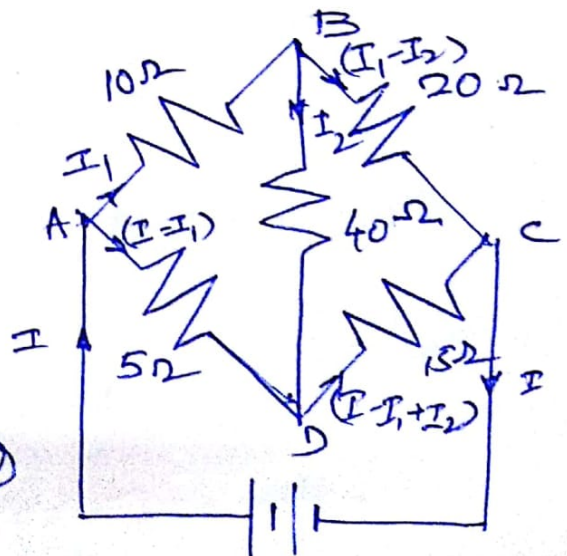
BCDB

$$-20(I_1 - I_2) + 15(I - I_1 + I_2) + 40I_2 = 0$$

$$-35I_1 + \cancel{75I_2} + 15I = 0 \quad \text{--- (2)}$$

ADEA

$$-5(I - I_1) - 15(I - I_1 + I_2) + 20 = 0 / 20I_1 - 15I_2 - 20I = -20 \quad 20V$$



$$D_2 = \begin{vmatrix} -15 & -40 & 5 \\ -35 & 75 & 15 \\ 20 & -15 & -20 \end{vmatrix} = \begin{matrix} +19125 \\ \cancel{25875} + 16,000 - \cancel{1025} \\ -4875 \end{matrix}$$

$$D_1 = \begin{vmatrix} 0 & -40 & 5 \\ 0 & 75 & 15 \\ -20 & -15 & -20 \end{vmatrix} = \begin{matrix} +12,000 + 7500 \\ = 19,500 \end{matrix} \quad I_1 = \frac{D_1}{D} = 0.644 \text{ A}$$

$$D_2 = \begin{vmatrix} -15 & 0 & 5 \\ -35 & 0 & 15 \\ 20 & -20 & -20 \end{vmatrix} = \begin{matrix} -4500 + \cancel{2000} 3500 \\ = -\cancel{2000} - 1000 \\ -0.033 \end{matrix}$$

$$\therefore I_2 = \frac{D_2}{D} = \cancel{0.078} \text{ A}$$

$$D_3 = \begin{vmatrix} -15 & -40 & 0 \\ -35 & 75 & 0 \\ 20 & -15 & -20 \end{vmatrix} = 22500 + 28000 = 50,500$$

$$I = \frac{D_3}{D} = 1.67 \text{ A}$$

Branch Current

AB $I_1 = 0.644 \text{ A}$

BC $I_1 - I_2 = 0.677 \text{ A}$

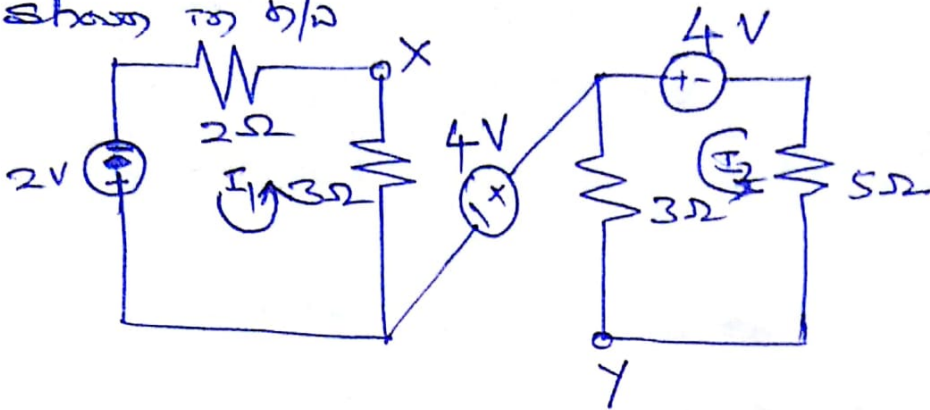
CD $I_1 - I_2 + I_3 = 1.06 \text{ A}$

DA $I - I_1 = 1.026 \text{ A}$

BD $I_2 = -0.033 \text{ A}$

Ex) Apply Kirchhoff's laws to find p.d. betⁿ a & y

shown in b/w



Solⁿ $I_1 = 2/5 = 0.4 \text{ A}$

$I_2 = 4/8 = 0.5 \text{ A}$

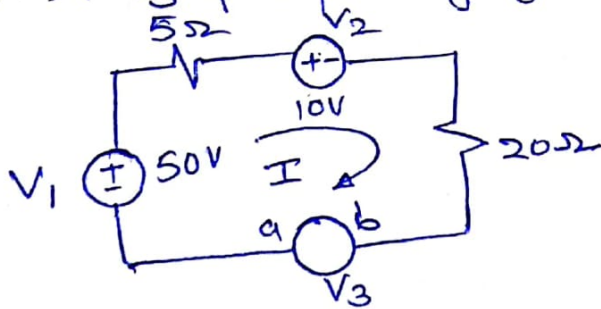
$V_4 = 1.2 \text{ V}$ $V_3 = 1.5 \text{ V}$

$V_{xy} = 1.2 \text{ V} + 4 \text{ V} - 1.5 \text{ V}$
 $= 3.7 \text{ V}$

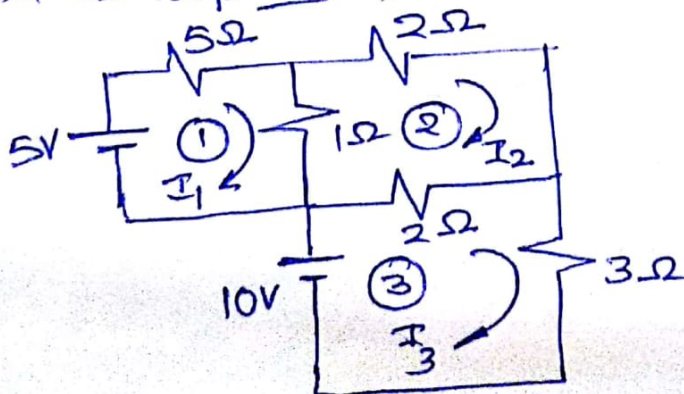
0.000266
 0.000004

Assignment -

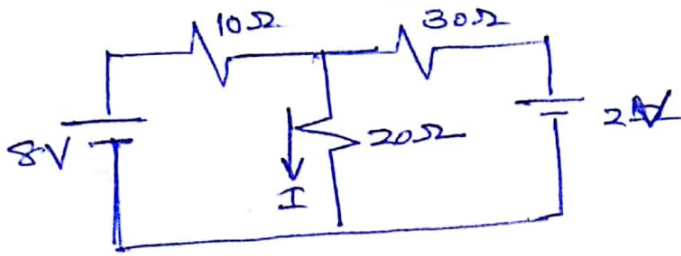
1) Find V_3 & its polarity if the ckt I in the ckt shown is 0.4 A



2) Find the loop cks in the ckt shown in fig.

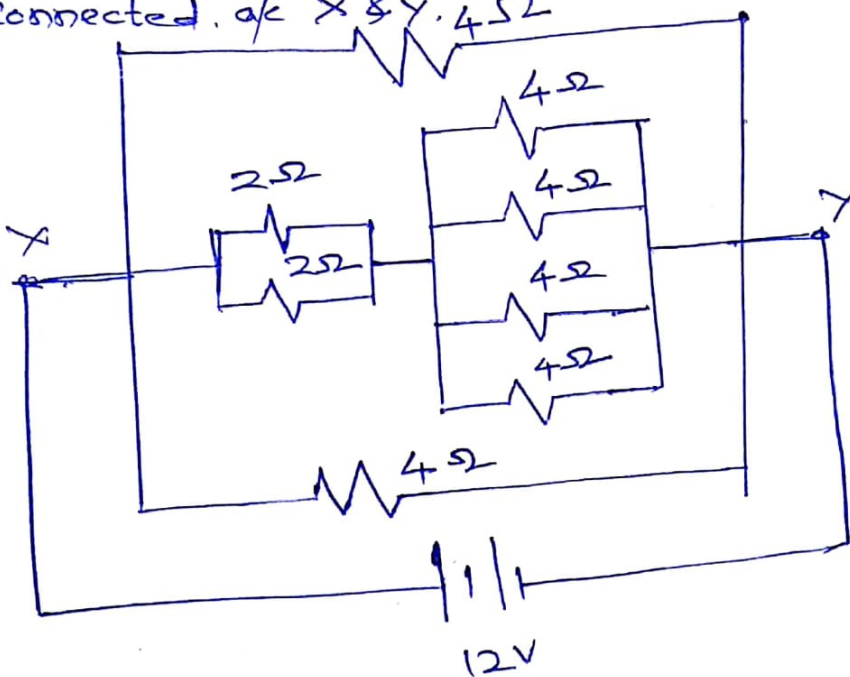


③ Find the ct through 20Ω in the ckt shown



$$I = 0.249A$$

④ Find the total resistance of the ckt a/c the terminals X & Y
 Also find the power consumed by the ckt if a 12V battery
 is connected, a/c X & Y. 4Ω



$$R_{xy} = 1\Omega$$

$$I = 12A$$

$$P = 144W$$

AC Fundamentals

- Adv. 1) The v_g can be raised or lowered with the help of a device called transformer. In dc s/m, raising or lowering a v_g is not so easy.
- 2) As the v_g can be raised, electrical power transⁿ at higher v_g is possible. Now, higher the v_g , lower is the ct flowing through the transmission line. Hence, losses are less & lesser conducting material is req. This makes ac transⁿ economical & efficient.
- 3) AC motors are simple in construction, very robust, cheaper & require less or no maintenance.
- 4) Whenever required AC can be easily converted into DC.

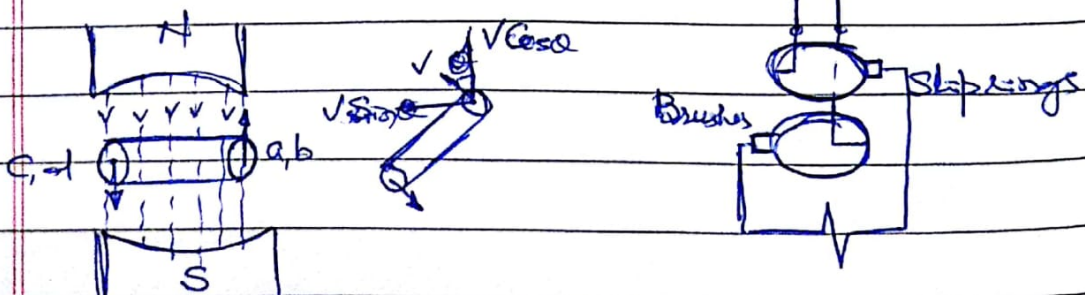
Generation of AC Voltage -

The m/c's which are used to generate elect. v_g s are called generators. The generators which are used to generate ac v_g are called as "Alternators".

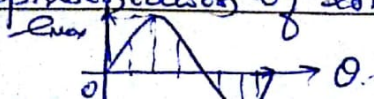
The basic principle of operation of an alternator is "Electro-magnetic Induction". It says, 'whenever there is a relative motion betⁿ the conductor & magnetic field, in which it is kept, an emf is induced in the conductor.'

Construction:

Winding -



Graphical Representation of induced EMF



AC Fundamentals

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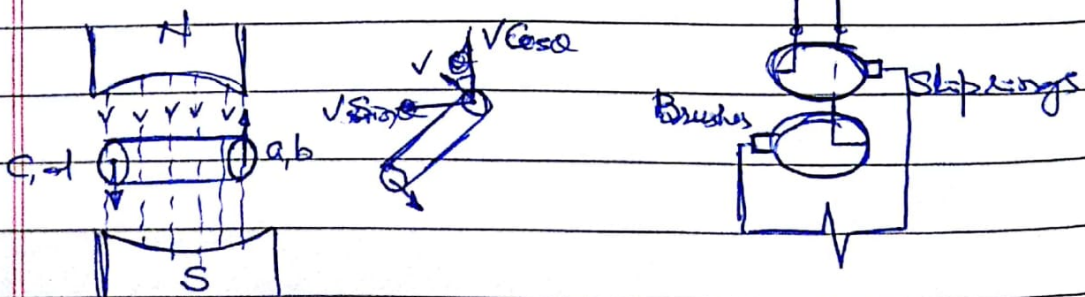
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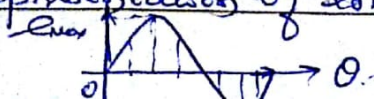
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Construction:

Winding -



Graphical Representation of induced EMF



Cycle - It is defined as a set of positive & negative instantaneous values of the alternating quantity.

Time period (T) - It is the time taken by an alternating quantity to complete its one cycle. Denoted by T & measured in sec.

Frequency - (f) - The no. of cycles completed by an alternating quantity per sec is its freq. Denoted by f , & measured in cycles/sec or Hz.

Equation of Generated EMF -

B - Flux density of mag. field in wb/m^2

l - Active length of each conductor in mts

r - radius of circular path traced by conductors

ω - angular velocity in rad/sec

v - linear velocity in m/s .

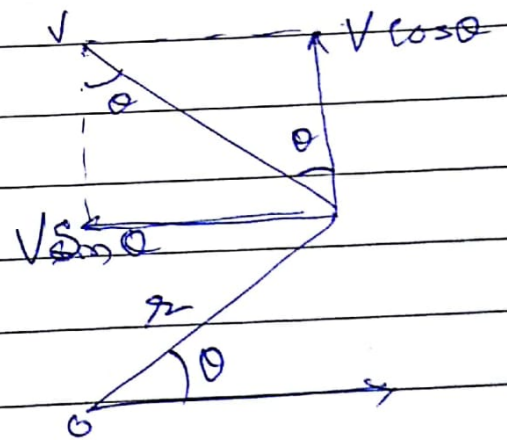
Now $v = r\omega$.

According to Faraday's law

$$e = Blv \sin \theta$$

$$e = E_m = Blv \text{ when } \theta = 90^\circ$$

$$\therefore e = E_m \sin \theta$$



Equation of an Alternating Quantity -

Let, B - flux density in wb/m^2

L - Active length of the conductor in m

r - Radius of circular path traced by the condⁿ in m

ω - Angular velocity of coil in rad/sec

v - Linear velocity in m/s

Now $v = r\omega$.

$\theta = \omega t$.

According to Faraday's laws of electromagnetic induction,

$$e = Blv \sin\theta$$

When $\theta = 90^\circ$ $\sin\theta = 1$

\therefore Max. emf induced is $E_m = Blv$

Hence the instantaneous value of generated emf is

$$e = E_m \sin\theta = E_m \sin\omega t.$$

Ex: An alternating v_g of time period 0.02 sec has max value of $12V$. Write the eqⁿ for its instantaneous value.

Calculate the inst. value of the v_g after time 0.002 sec, where v_g is taken from the inst. of zero v_g & is becoming +ve

Also, calculate the time req. for the v_g to reach $4V$ for the first time.

Solⁿ - Given data: $T = 0.02 \text{ sec}$ $E_m = 12 \text{ V}$

$$\begin{aligned} \text{WKT } e &= E_m \sin \omega t = E_m \sin \frac{2\pi t}{T} \\ &= 12 \sin(100\pi t) \text{ V} \end{aligned}$$

$$\begin{aligned} \text{at } t = 0.002 \text{ sec, } e &= 12 \sin(100 \times \pi \times 0.002) \\ &= 7.053 \text{ V.} \end{aligned}$$

Now, the inst. value is $e = 4 \text{ V}$

$$\therefore 4 = 12 \sin(100\pi t)$$

$$\sin(100\pi t) = \frac{4}{12}$$

$$\therefore t = 1.084 \times 10^{-3} \text{ sec.}$$

Effective value: (RMS Value)

An alternating quantity varies from instant to instant, while the direct ct is const. w.r. to time. So, for the comparison of two, there must be some common platform. Such platform can be the effect produced by the two cts. One of the such effects is the heating of resistance, due to ct passing through it.

"The effective or rms value of an alternating ct is given by that steady ct (DC) which when flowing through a given ckt for a given time, produces the same amount of heat as produced by the alternating ct, which when flows through the same ckt for the same time.

RMS Value -

Consider sinusoidally
varying alternating current
& square of this current

$$\text{Current } i = I_m \sin \theta \text{ where}$$

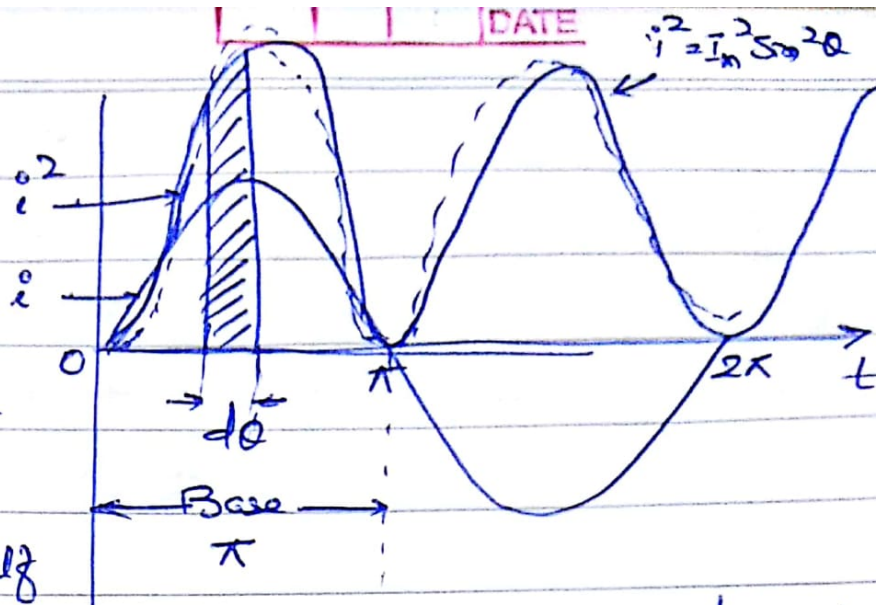
$$i^2 = I_m^2 \sin^2 \theta$$

Area of curve over a half

cycle can be calculated by considering an interval $d\theta$ as shown

Area of square curve over half cycle

$$= \int_0^\pi i^2 d\theta \text{ \& the length of the base is } \pi$$



\therefore Avg value of square of the current over half cycle

$$= \frac{\text{Area of curve over half cycle}}{\text{Base over half cycle}} = \frac{\int_0^\pi i^2 d\theta}{\pi}$$

$$= \frac{1}{\pi} \int_0^\pi I_m^2 \sin^2 \theta \cdot d\theta = \frac{I_m^2}{\pi} \int_0^\pi \left[\frac{1 - \cos 2\theta}{2} \right] d\theta$$

$$= \frac{I_m^2}{2\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^\pi = \frac{I_m^2}{2\pi} (\pi)$$

$$= \frac{I_m^2}{2}$$

\therefore Root Mean Square Value can be calculated as

$$I_{RMS} = \sqrt{\text{Avg. of square of current}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\therefore I_{RMS} = 0.707 I_m$$

Importance of RMS Value -

1. In case of a.c., rms value specifies magnitudes of a.c.
2. The ammeters, voltmeters record rms values of ct & vg.
3. The heat produced due to ac is proportional to square of the rms value of the ct.

2. Average Value -

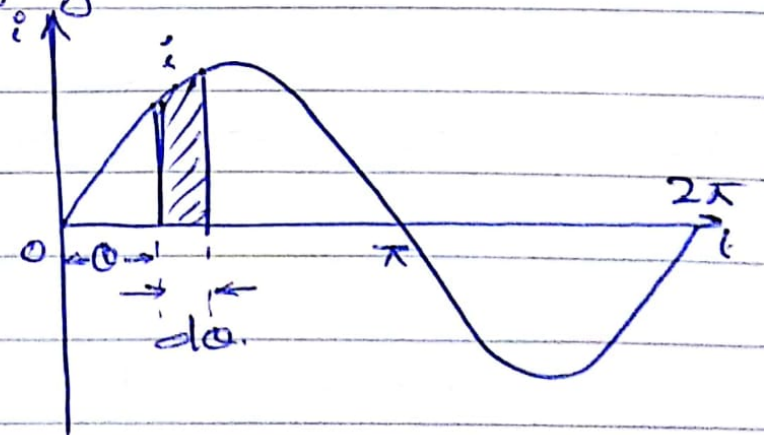
It is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

For an unsymmetrical ac, the avg value must be obtained for one complete cycle but for symmetrical ac, it is to be obtained for half cycle.

C₁, sinusoidally varying
ct $i = I_m \sin \theta$

C₂, a small interval of $d\theta$

The avg inst. value of ct in this interval is say i' .



The avg value can be obtained by taking ratio of area under curve over half cycle to the base for half cycle.

$$\begin{aligned}
 \therefore I_{av} &= \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}} \\
 &= \frac{\int_0^{\pi} i \, d\theta}{\pi} \\
 &= \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta = \frac{I_m}{\pi} (-\cos \theta) \Big|_0^{\pi} \\
 &= \frac{I_m}{\pi} [-\cos \pi + \cos 0] \\
 &= \frac{2I_m}{\pi}
 \end{aligned}$$

$$\therefore I_{av} = 0.637 I_m$$

Importance of Avg. value -

1. The avg value ~~is~~ indicates dc quantity.
2. The avg values of v_g & e_c play an imp. role in analysis of the rectifier ckts.
3. The charge transferred in capacitor ckts is measured using avg values.

Form Factor (Kf)

It is the ratio of rms value to the avg value

$$K_f = \frac{\text{rms value}}{\text{avg value}} = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

Crest (Peak) Factor: (k_p)

It is the ratio of max. value to the rms value of a c quantity

$$k_p = \frac{I_m}{0.707 I_m} = 1.414.$$

Ex: An alternating v_g has an effective value of 70.7106 V & f of 60 Hz. Find its avg value, rms factor, crest factor assuming it to be purely sinusoidal.

Sol: Given data: $E_{rms} = 70.7106 \text{ V}$, $f = 60 \text{ Hz}$

$$E_m = 1.414 \times 70.7106 = 100 \text{ V} \quad \text{or} \quad \sqrt{2} E_{rms} = 100 \text{ V}.$$

$$E_{av} = 0.637 E_m = 63.7 \text{ V}$$

$$k_f = \frac{70.7106}{63.7} = 1.11 \quad \& \quad k_p = \frac{100}{70.7106} = 1.414$$

Ex: An alternating ct of f 60 Hz has a max. value of 12 A

(i) Write the eqⁿ for inst. values. (ii) Find the value of ct after $1/360$ sec (iii) Time taken to reach 9.6 A for the first time.

In the above cases assume that time ref. is zero when ct wave is passing through zero & increasing in the +ve direction.

Sol: $f = 60 \text{ Hz}$, $I_m = 12 \text{ A}$ $\omega = 2\pi f = 377 \text{ rad/sec}$

$$(i) \quad i = I_m \sin \omega t = 12 \sin 377 t$$

$$(ii) \quad t = 1/360 \quad \therefore i = 12 \sin 377 / 360 = 10.3924 \text{ A}$$

$$(iii) \quad i = 9.6 \text{ A} \quad \therefore 9.6 = 12 \sin 377 t \quad \therefore t = 2.459 \text{ sec}$$

Ex: For the ct wave shown in fig. Find (i) Peak ct

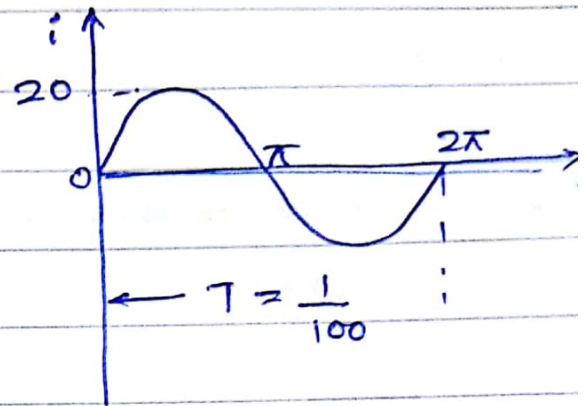
(ii) Avg value (iii) Freq. (iv) Periodic time. (v) Inst. value at ~~3ms~~

$$t = 3 \text{ ms.}$$

Solⁿ Given data:

$$I_m = 20 \text{ A}$$

$$T = \frac{1}{100} \text{ sec}$$



(i) Peak value $I_m = 20 \text{ A}$

(ii) Avg value $I_{avg} = 0.637 \times 20 = 12.73 \text{ A}$

(iii) Freq. $f = \frac{1}{T} = \frac{1}{1/100} = 100 \text{ Hz}$

(iv) Periodic time $T = \frac{1}{100} = 0.01 \text{ sec}$

(v) Inst. value at $t = 3 \text{ ms} = I_m \sin \omega t$

$$\therefore i = 20 \sin (2\pi f \times 3 \times 10^{-3})$$

$$= 19.02 \text{ A.}$$

Ex: An alternating ct has an effective value of 200 A .

If its freq. is 25 Hz , find its avg value. Write the

expression for the ct

Phasor Representation of an Alternating Quantity -

In the analysis of ac cts, it is very difficult to deal with ac quantities in terms of their waveforms & mathematical eqns. ~~The~~ Adding, subtracting etc of the two alternating quantities is tedious & time consuming in terms of their mathematical eqns. Hence it is necessary to study a method which gives an easier way of representing an alternating quantity. Such a representation is called "Phasor Representation" of an alternating quantity.

In this method,

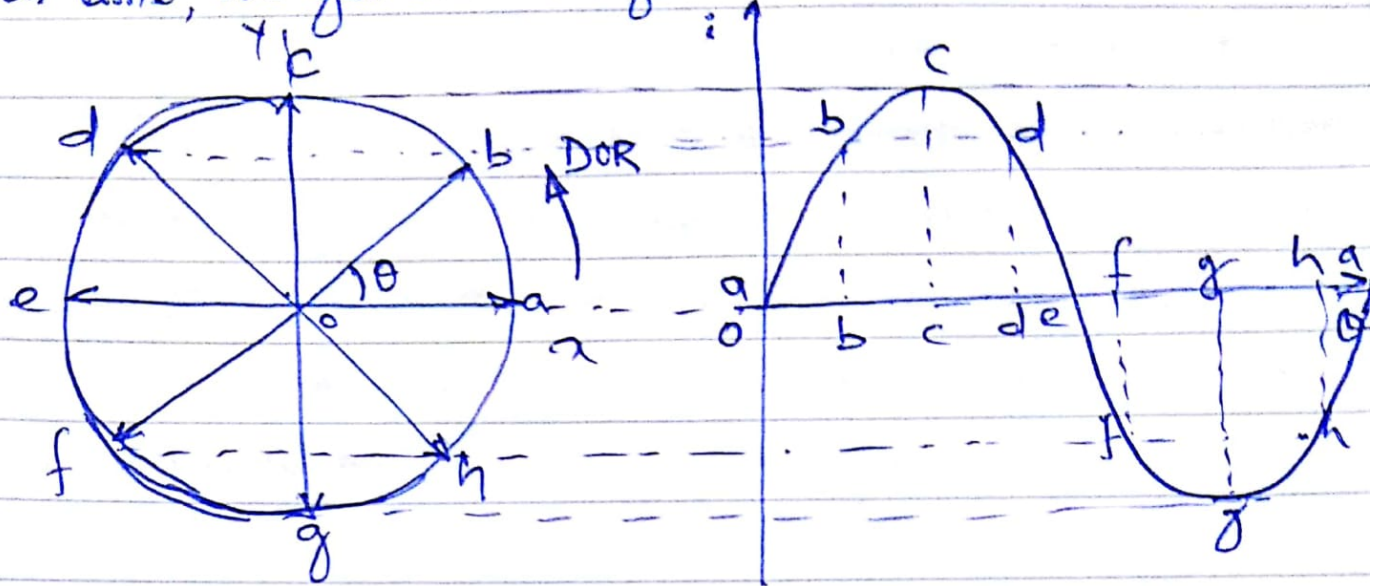
The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow. The length of the line represents the magnitude of an alternating quantity & arrow indicates the direction. This is similar to vector representation. Such a line is called 'Phasor'.

['The phasors are assumed to be rotated in anticlockwise direction'.]

One complete cycle of a sine wave is represented by one complete rotation of phasor.

Ex, a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting point O . If we project this phasor on Y -axis ~~eg~~ & plot against the angle'

' θ ' vs time, we get sine waveform.



∴, various positions as shown,

1. At pt 'a', the Y-axis projection is zero. The inst. value of e is also zero.
2. At pt 'b', the Y-axis projection is $Ob \sin \theta = r \sin \theta$. The length of the phasor is equal to max. value of an alternating quantity. So, inst. value of e at this position is $i = I_m \sin \theta$.
3. At pt. 'c', the Y-axis position is OC & it represents the entire length of the phasor i.e. the inst. value represents its max. value.
4. At pt. 'd', Y-axis projection becomes $I_m \sin \theta$ which is the inst. value of e at that inst.
5. At pt. 'e', the Y-axis projection is zero, & inst. value of e is also zero at this inst.
6. Similarly, at pts, 'f', 'g' & 'h', the Y-axis projections give us inst. values of e at resp. insts & when plotted give us -ve cycle of alternating quantity.

Thus, if the length of the phasor is taken equal to the max. value of the alternating quantity, then its rotation in space at any inst. is such that the length of its projection on the Y-axis gives the inst. value of the alternating quantity at that particular inst. The angular velocity ' ω ' of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle. i.e. $\theta = \omega t$, $\omega = 2\pi f$.

In practice, the alternating quantities are represented by their rms values. Hence, the length of the phasor represents rms value of the alternating quantity.

Phasors are always assumed to be rotated in anti-clockwise direction.

Two alternating quantities of same freq. can be represented on same phasor diagram.

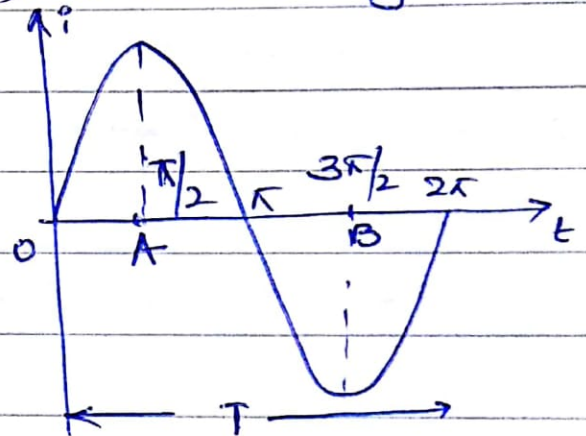
If freq. of two alternating quantities are diff., then such quantities cannot be represented on the same phasor diagram.

Phase -

An alternating quantity changes its magnitude & direction at every inst. So, it is necessary to know the condⁿ of the alternating quantity at a particular instant. The location of the condⁿ of the alternating quantity at any particular instant is called its phase.

Phase may be defined as 'the alternating quantity at any particular the fractional part of a period through which the quantity has advanced from the selected origin.'

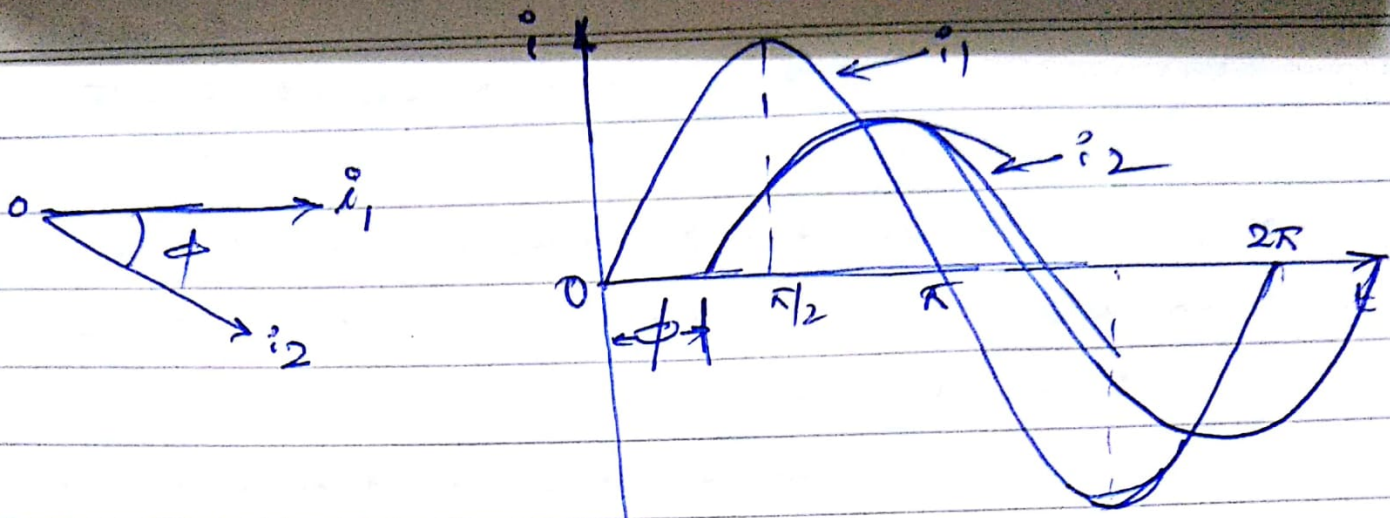
e.g. - phase of ct at pt. 'A' is $\pi/4$ Sec, or $\pi/2$ rad.



Phase difference - (Lagging or Leading of Sinusoidal wave)

When two alternating quantities, say two vgs or two cts or one vgs & one ct are considered simultaneously, the freq. being the same, they may not pass through a particular point at the same instant.

One may pass through a max. value at the instant when the other passes through a value other than its max. value. These two quantities are said to have a phase difference. Phase diff. is specified either in degrees or in radians.



The quantity ahead in phase or which reaches its max. value first is said to lead the other quantity, whereas the other quantity is said to lag behind the first quantity.

If i_1 is taken as ref. then i_2 lags i_1 by an angle ϕ .

If i_2 is taken as ref. then i_1 leads i_2 by an angle ϕ .

$$i_1 = I_{m1} \sin \omega t, \text{ then } i_2 = I_{m2} \sin (\omega t - \phi)$$