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Module -2

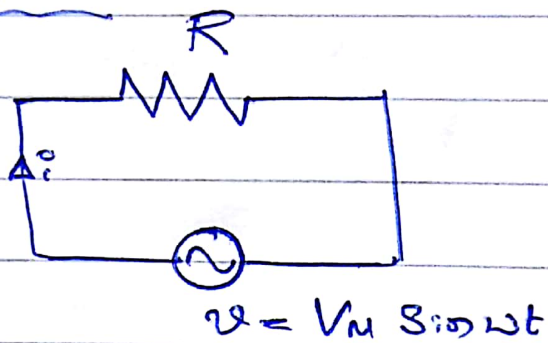
Single Phase AC Circuits —

The resistance, inductance & capacitance are the three basic elements of any elect. n/w. In order to analyse any elect. ckt, it is necessary to understand

1) AC through pure resistive ckt
2) — " — " — inductive ckt
3) — " — " — Capacitive ckt

1. AC through pure Resistive Circuit —

Cn, a pure resistive ckt with a resistance of R ohms connected across a sinusoidal vg. v given by



$$v = V_M \sin \omega t \quad — (1)$$

By ohms law, instantaneous ct in the ckt is

$$i = \frac{v}{R} = \frac{V_M \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t$$

$$= I_m \sin \omega t. \quad — (2)$$

where $I_m = V_m / R$.

Comparing eq^ns (1) & (2), it is clear that vg & ct are in phase with each other. ie phase diff. bet^n them is zero

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Power:

Inst. power $p = v \times i$.

$$= V_m \sin \omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{(1 - \cos 2\omega t)}{2}$$

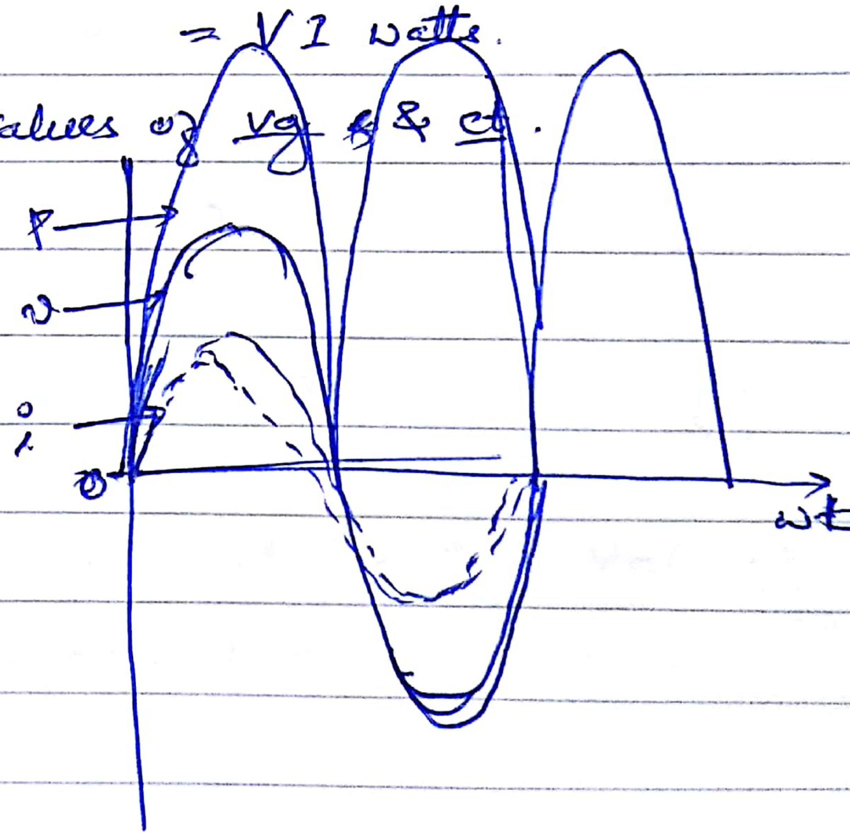
$$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}$$

For a one complete cycle, the avg. value of $\frac{V_m I_m}{2} \cos 2\omega t$ is zero.

$\therefore$ Avg. power for whole cycle. $P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

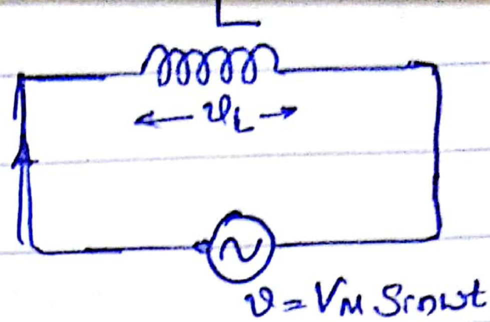
$= VI$ watts.

Where V & I are RMS values of vg & ct.



AC through a purely Inductive Circuit —

Cr, an ac ckt. consisting of a pure inductance of L Henry. Let the inst. value of applied vg is $v = V_M \sin\omega t$ — (1)



This ac vg will cause an ac ct 'i' to flow through the ckt. Due to the inductance of the coil, a self induced emf is induced in the coil. Which opposes the applied vg at every instant.

$$\therefore \quad v_L = -L\frac{di}{dt}$$

By applying KVL

$$v - v_L = 0 \quad \text{or} \quad V_M \sin\omega t = L\frac{di}{dt}$$

$$\therefore \quad \frac{di}{dt} = \frac{V_M}{L}\sin\omega t$$

Integrating both sides,

$$\int \frac{di}{dt} = \int \frac{V_M}{L}\sin\omega t$$

$$\text{or} \quad i = \frac{V_m}{L}\left(-\frac{\cos\omega t}{\omega}\right)$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_M}{\omega L}\sin(\omega t - \pi/2)$$

Ct is max. when $\sin(\omega t - \pi/2) = 1 \quad \therefore I_m = V_m/\omega L = \frac{V_M}{X_L}$

where X_L is inductive reactance $= \omega L$ measured in Ohms

$$\therefore \quad i = I_m \sin(\omega t - \pi/2) \quad \text{— (2)}$$

Comparing eq^n ① & ⑤ ~~we get~~ it is clear that i_L always lags the applied vg by an angle 90° or $\pi/2$ rad.

Power

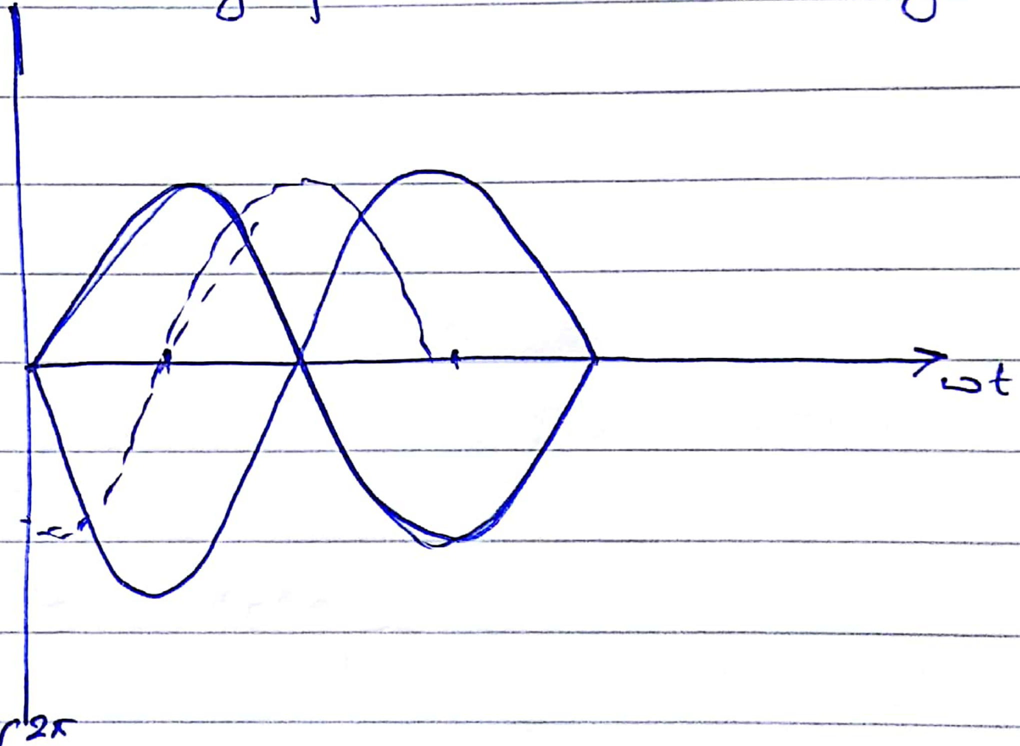
Instantaneous power $p = v \times i$

$= V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$

$= -V_m I_m \sin\omega t \cdot \cos\omega t$.

$= -\frac{V_m I_m}{2} \sin 2\omega t$.

The avg power consumed by a pure inductive ckt over a cycle is zero.

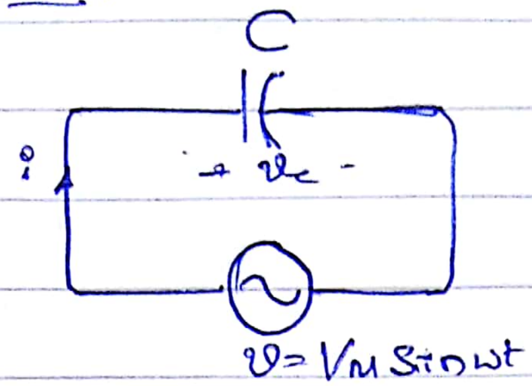


$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t$$

$$= -\frac{V_m I_m}{4\pi}\left(-\frac{\cos 2\omega t}{2}\right)\Big|_0^{2\pi}$$

$$= 0$$

AC through pure Capacitive Ckt.:



Let $v = V_M \sin \omega t$ be the applied vg. a/c a capacitor of Capacitance ~~to~~ C Farads

Inst. Charge $q = Cv = C V_M \sin \omega t$

Capacitor ct is equal to the rate of change of charge

or $i = \frac{dq}{dt} = \frac{d}{dt} C V_M \sin \omega t$

$$= \omega C V_M \cos \omega t$$

$$= \frac{V_M}{1/\omega C} \cos \omega t$$

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$$= \frac{V_M}{X_C} \cos \omega t = I_M \sin(\omega t + \pi/2)$$

$$X_C = 1/\omega C = 1/2\pi f C$$

In pure capacitive ckt ct leads vg. by $\pi/2$ or $90°$

Power $p = v \times i$

$$= \frac{V_M I_m}{2} \cos 2\omega t$$

Avg. power over a full cycle is zero.

$P = 0$ Hence, power consumed by a pure capacitance in a ac ckt is zero.

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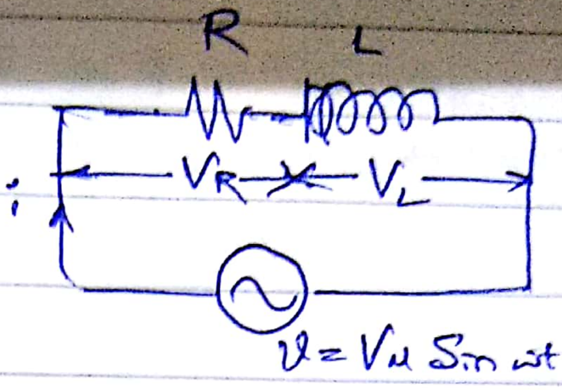
AC through R-L Circuit

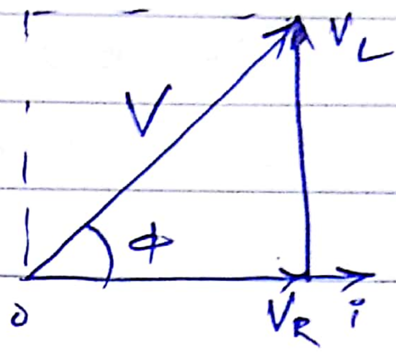


$$\bar{V} = \bar{V_R} + \bar{V_L}$$

$$\bar{V} = \bar{IR} + \bar{IX_L}$$

$$\bar{IZ} = \bar{IR} + \bar{IX_L}$$

$$\text{or } \bar{Z} = R + jX_L$$



From vector diagram

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\text{or } Z = \sqrt{R^2 + X_L^2}$$

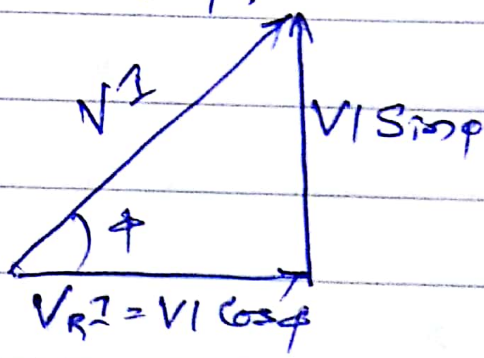
$$\& \quad i = I_m \sin(\omega t - \phi)$$

Power — $P = V_R I = V\cos\phi\, I = VI\cos\phi.$

$$\bar{V} = \bar{V_R} + \bar{V_L}$$

$$\bar{VI} = \bar{V_R I} + \bar{V_L I}$$

$$= \bar{VI\cos\phi} + \bar{VI\sin\phi}$$



These three terms are defined as

(1) Apparent Power (S)

It is the product of rms values of vg (V) & ct (I)

& is denoted by $\boxed{S = V \times I.\ VA}$

It is measured in volt-amp (VA) or kilo-volt-amp (KVA)

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(2) Real or True or Active Power (P):

It is the product of applied vg & active component of the ct. It is measured in watts (W) or kilo watts (kW)

$$\boxed{P = VI\cos\phi \text{ watts}}$$

(3) Reactive Power (Q):

It is the product of applied vg & reactive component of the ct. Its unit is Volt-ampere-reactive (VAR).

$$\boxed{Q = VI\sin\phi \text{ VAR}}$$

Power Factor:

It is defined as a factor by which the apparent power must be multiplied in order to obtain true power.

It is the ratio of ~~apparent~~ true power to ~~true~~ apparent power

$$\text{Power Factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi$$

It is the cosine of the angle between applied vg & ct drawn from the supply. It can not be greater than 1.

It is also defined as ratio of resistance to impedance

$$\cos\phi = R/Z$$

The nature of p.f is always determined by position of ct w.r.to vg leading or lagging.

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Ex: A 100V, 50 Hz inductive ckt takes a ct of 10A lagging the vg by 30°. Calculate the resistance & inductance of the ckt.

Given: $V = 100V$, $f = 50Hz$, $I = 10A$, $\phi = 30^\circ$

$\therefore I = 10\angle -30^\circ$ A.

$\therefore Z = \frac{V}{I} = \frac{100\angle 0}{10\angle -30} = 10\angle 30\ \Omega$.

$= 8.66 + j5\ \Omega$

$\therefore R = 8.66\Omega$ & $X_L = 5\Omega$

$2\pi f L = 5\Omega$ $\therefore L = 0.0159$ H.

Ex: A series R-L ckt takes 400W at a p.f of 0.8 from a 120V, 50Hz supply. Calculate the values of R & L.

Soln: Given: $P = 400W$, $f = 50Hz$, $V = 120V$, $\cos\phi = 0.8$

$P = VI\cos\phi$

$400 = 120 \times I \times 0.8$ or $I = 4.167$ A

$\therefore Z = V/I = 28.8\Omega$

WKT $\cos\phi = R/Z$ or $R = Z\cos\phi = 23.04\ \Omega$

& $\tan\phi = X_L/R$ or $X_L = R\tan\phi = 17.27\ \Omega$

$X_L = 2\pi f L = 17.27\Omega$

$\therefore L = 0.055\Omega$

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Ex: An emf of $v = 220 \sin(314t - 1.5708)$ V is applied to a series ckt. The ct flowing is $i = 20 \sin(314t - 10°)$ A. Find the freq. & the values of ckt elements.

Soln: $v = 220 \sin(314t - 1.5708)$ V

$\therefore V_m = 220$ V & $\phi = -1.5708$ rad $= -90°$

$\omega = 314 \quad \therefore f = 50$ Hz

$i = 20 \sin(314t - 10°)$

$\therefore I_m = 20$ A & $\phi = -10°$

$V = \dfrac{V_m}{\sqrt{2}} = 155.56$ V & $I = \dfrac{I_m}{\sqrt{2}} = 14.142$ A

$\therefore V = 155.56 \angle -90°$ V & $I = 14.142 \angle -10°$ A

$\therefore Z = \dfrac{V}{I} = 11 \angle -80° \ \Omega$

$= 1.91 - j10.832 \ \Omega$

$\therefore R = 1.91 \ \Omega$ & $X_C = 10.832 \ \Omega$

$\therefore C = 2.93 \times 10^{-4}$ F

Asgn

Ex: An alternating vg of $(160 + j120)$ V is applied to a ckt. & the ct is given by $(6 + j8)$ A. Find the values of ckt elements for 50 Hz freq. Also calculate power factor of the ckt & power consumed.

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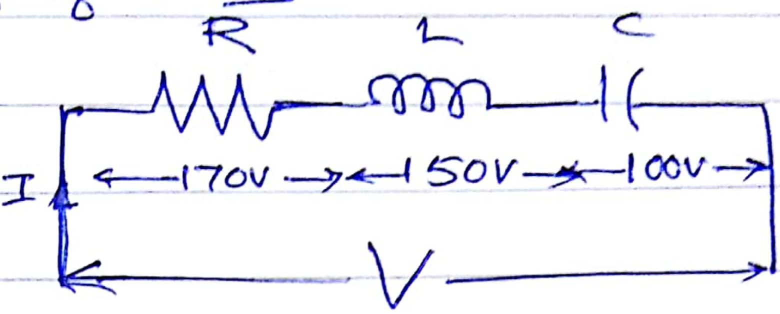
Ex: A certain vg is applied to a series R-L-C ckt. The vg a/c these elements are 170V, 150V, & 100V resp. The ct drawn is 4A. Find the p.f. of the ckt.



Soln

$V_R = IR$

or $R = 42.5\,\Omega$

$V_L = I X_L$

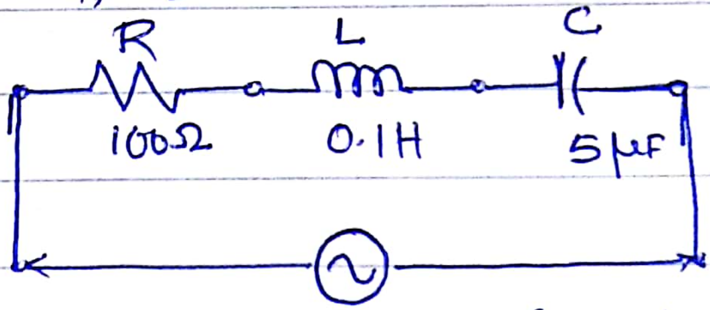
$\therefore X_L = 37.5\,\Omega$ $\quad V_C = I X_C$ $\quad \therefore X_C = 25\,\Omega$

$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2} = 44.3\,\Omega$

$\cos\phi = R/Z = 0.9593$. lag.

Lagging because $X_L > X_C$

Ex: A series RLC ckt has $R = 100\,\Omega$, $L = 0.1\,H$ & $C = 5\,\mu F$. A vg $v(t) = 141.4 \cos 377t$ V is applied to the ckt. Det. the ~~vg~~ ct, & vgs of V_R, V_L & V_C.



Soln: $v(t) = 141.4 \cos 377t$

or $= 141.4 \sin(377t + 90^\circ)$

$\therefore V_m = 141.4$, $\omega = 377$

$\therefore V = V_m/\sqrt{2} = 100\,V$

$V = 100 \angle 90^\circ\,V$ $\quad X_L = \omega L = 377 \angle 90^\circ\,\Omega$, $X_C = 530.5 \angle -90^\circ$

$Z_T = R + X_L + X_C = 183.20 \angle -56.917^\circ\,\Omega$

$I = V/Z_T = 0.5458 \angle 146.917$ A

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$\therefore \; V_R = I \times R = 54.58 \angle 146.917 \; V$

$V_L = I \times X_L = 205.766 \angle 236.917 \; V$

$V_C = I \times X_C = 289.549 \angle 56.917 \; V$

$\therefore \; |V_R| = 54.58V, \quad |V_L| = 205.766V \quad |V_C| = 289.549V$